

Supplementary Materials

Characterizing the Jaynes-Cummings Dimer Characterizing the system involves locating the two qubit flux bias points required for (1) efficient initialization and (2) strong photon-photon interactions during dynamics. Since both qubits are individually tunable the system is characterized by two independent voltages V_L and V_R . The conventional method for mapping out circuit-QED systems is to look at the frequency response in transmission at low drive power as a function of flux bias voltages. This works because the low-lying polariton mode frequencies are sensitive to qubit detunings, and allow direct measurement of the spectra.

Our early physical realizations of dimers with relatively high dissipation rates ($\kappa \approx 10$ MHz) were successfully characterized in this manner. Figure (S1) shows a map of an older device as a function of V_L and V_R . Here drive frequency is fixed and transmission through the dimer is monitored, producing a large signal only when a first excitation manifold polariton mode with a significant resonator component shifts to be resonant with the drive frequency. A frequency far from ν_c is chosen, highlighting regions of flux space near resonance. The map shows the periodicity due to flux quantization, with the non-orthogonality of lattice vectors stemming from cross-coupling of the flux biases. The indicated contour on figure (S1) brings both qubits through resonance, one from above ($\nu_a^L > \nu_c$) and one from below ($\nu_a^R < \nu_c$). Transmission as a function of drive frequency along this contour is shown in figure (S2). At resonance, the four polariton modes are clearly visible at the expected frequencies ($\nu_c \pm g \pm J/2$).

Calibration using this method has proven challenging for later devices fabricated with very low dissipation, important for observing long timescale dynamics. Polariton modes with a significant qubit component (true for all modes in resonance) are particularly difficult to measure, as dissipation channels not contributing to measurement reduce the signal to noise ratio (SNR). This difficulty is compounded by the presence of multiple cavities, with the end result that low power characterization of low dissipation dimers has proven impractical. A new technique is required for rapid mapping of flux space.

The bistability due to the Jaynes-Cummings nonlinearity can be used to probe the spectrum quickly and with large SNR. The technique is similar to the high fidelity qubit state readout technique of Reed et al. [1] (see also [2, 3]). In their experiment, a difference in cavity frequency when the qubit is in the ground or excited state ($\nu \approx \nu_c \pm \frac{g^2}{\Delta} \sigma_z$, where Δ is the detuning of the

qubit from the cavity) creates a qubit state dependent threshold power, which is the drive power required to enter the bright state when the cavity is driven by a pulse at the bare resonator frequency. Setting a pulse power between the two threshold powers leads to high distinguishability between the ground and excited state of the qubit.

For the dimer, we take advantage of the fact that the threshold behavior is sensitively dependent on the proximity of low-lying polariton modes to the linear dimer modes (where qubits are decoupled but the resonator coupling leads to a symmetric (ν_S) and an antisymmetric (ν_A) mode, where $\nu_{S,A} = \nu_c \pm J$, and the lower energy mode is the symmetric one). Transmission through the dimer is monitored while a continuous microwave drive ($\nu_d = \nu_{S,A}$) is swept in power. Very little averaging is required to distinguish the dark and bright state, greatly speeding up the time required to perform a characterization.

Plotting threshold power as a function of the two flux biases produces maps like the one shown in figure (S3), which is for the antisymmetric mode. Threshold power varies by over two orders of magnitude, making it easy to pick out important features. The blue regions are immediately noticeable and can be identified with minimum qubit energies leading to incredibly weak nonlinearity. This weak nonlinearity is ideal for loading photons into the dimer, and is used during initialization. Additionally, locating these reference points simplifies the search for resonance by restricting the space needed to be searched to a contour in flux space connecting the $(\Phi_L, \Phi_R) = (-1/2, -1/2)$ and $(\Phi_L, \Phi_R) = (1/2, 1/2)$, with flux expressed in units of the superconducting flux quantum $\Phi_0 = h/2e$. Since both qubits are nearly identical, this contour simultaneously moves both qubits from their minimum energies to their maximum energies, passing through resonance together.

This contour is shown in figure (S4) for both the antisymmetric (a) and symmetric (b) modes. Averaged transmission is shown as a function of power and flux, along a contour in two-dimensional flux space, revealing the full structure of the transition from dark state into bright state, beyond just a simple threshold power. The detailed shape of the transition region depends on the specific settings of the power ramp (sweep time, starting power, IF bandwidth) likely stemming from the effect these settings have on the time spent at different powers and hence different probabilities of transition. In addition to broad shifts of the threshold power which we attribute to movement of the lowest excitation manifold, a good deal of fine structure is apparent,

thought to be due to the movements of the many states in higher excitation manifolds. When a higher excitation manifold state lines up with $\omega_{S,A}$, multi-photon transitions become important, despite their low probability. The precise details of this behavior are not fully understood at this time, but will be the subject of further work.

In the dimer system used throughout the paper, qubit maximum energies were built to be just above resonance with the cavities. This was important for reducing sensitivity to flux noise, but made locating the exact resonance point challenging. Resonance was identified as a point of very high nonlinearity (high threshold power) that simultaneously displayed a high degree of symmetry between the transition shape of the symmetric and antisymmetric dimer modes. For this experiment, a flux bias of $(\Phi_L, \Phi_R) = (-0.0286, -0.0286)$ was chosen as our resonance bias point.

Toy Model Showing Super-exponential Decay In this section we present a simplified toy model which captures the essence of the super-exponential decay as photon leakage takes the system from a region of Josephson oscillations through the cross-over and into the localized regime, presupposing the existence of the two regimes and their dynamical features.

In this model we average over a large number (10^6) of individual trials, each trial accounting for photon leakage probabilistically. All trials begin with an initial number of photons N_i in the dimer, each of which escapes according to a Poisson process with decay rate $\kappa = 225$ kHz. So long as the photon number at a given time is above the critical number for the transition ($N_c = 20$ for this simulation), the system is assumed to undergo linear oscillations at frequency $2J$, with a homodyne signal amplitude that scales with the photon number. When the photon number for the single trial crosses the critical threshold, the oscillations of that trial are extinguished, setting the amplitude to zero. The stochastic nature of photon leakage leads to the vanishing of the oscillation at a random time in each trial. Averaging over all trials in the ensemble produces the expected super-exponential decay signature, as shown for representative initial photon numbers in figure (S5a). In figure (S5b) we plot the toy model's output as a function of N_i and time, which produces an analog of the phase diagram appearing as figure (4c) in the main text.

In the actual experiment, as the region of the cross-over is approached in a single trial, the oscillations exhibit a critical slowing down, which appears as a randomization of the relative oscillation phase across trials, and hence an averaging (across all trials in an ensemble) to zero

of the oscillations. Additionally, when an individual trial has localized, the quantum collapse and revival leads to a rapid disappearance of the homodyne signal. When further account is taken, within the toy model, of the critical slowing down of the oscillation frequency, the phase boundary is shifted toward a higher critical photon number.

Driven Dynamics In nearly all experiments shown in this paper the initialization procedure involves a high-power drive applied when the qubits are far detuned. This ensures that the system is well above the Jaynes-Cummings threshold for linear system behavior during initialization. It is also important to note that the length of the drive pulse (typically $3/J$) is also short compared to the time it takes for the system to reach a steady state where drive and dissipation balance. Keeping the drive short ensures the initial photon number is a linear function of drive power, which is necessary for the calibration using the localized revival time behavior to be applicable at higher drive powers (see the discussion of figure (3a) in the main text). Unfortunately such short drive pulses limited the maximum initial photon number to about 500. To linearly delay the dissipation-induced transition it is necessary to exponentially increase N_i , which was accomplished using a much longer initialization procedure in figure (4b) in the main text ($11.5\mu s$ or $100/J$).

Figure (S6) shows the same data as in figure (4b) but includes the driven dynamics observed during the initialization period before photon-photon interactions were turned on by tuning the qubits into resonance. For drive powers above the Jaynes-Cummings threshold (-5 dBm) the system behaves as expected: super-exponential decay sets in at a time logarithmically dependent on the initial photon number. For lower powers, however, the driven dynamics show some interesting features due to the finite nonlinearity of the dispersive spectrum at low photon numbers. Josephson oscillations that build up over the first few microseconds of drive disappear, followed by a reappearance of the homodyne signal microseconds later. The full account of the driven-dissipative dynamics is beyond the scope of this work, and will require further study.

Simulation of Jaynes-Cummings Lattices on Classical Computers We briefly comment here on the difficulty of simulating the physics of Hamiltonians involving Hilbert spaces built as tensor products of many subsystems. We recall here the dimer Hamiltonian from the main body of the paper

$$\hat{H}_{dimer} = \hat{H}_L^{JC} + \hat{H}_R^{JC} - J \left(\hat{a}_L^\dagger \hat{a}_R + \hat{a}_R^\dagger \hat{a}_L \right), \quad (1)$$

with single site Jaynes-Cummings Hamiltonian being

$$\hat{H}^{JC} = \nu_c \hat{a}^\dagger \hat{a} + \nu_a \hat{\sigma}^+ \hat{\sigma}^- + g (\hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a}^\dagger) . \quad (2)$$

The dimension of the Hilbert space $\mathcal{H}$ necessary for simulating the Hamiltonian (1) scales as $\dim \mathcal{H} \sim (2\eta)^2$, with η the cutoff on the photon Fock space. For a lattice of n Jaynes-Cummings sites, this becomes $\dim \mathcal{H} \sim (2\eta)^n$, showing an exponential scaling of the Hilbert space size, and making classical simulations intractable for even quite small systems. This Hamiltonian conserves total excitation (polariton) number $\hat{\vartheta} = \sum_{s=L/R} (\hat{N}_s + \hat{\sigma}_s^+ \hat{\sigma}_s^-)$ ($\hat{N}_s$ is the total photon number operator on site s , while $\hat{\sigma}_s^+ \hat{\sigma}_s^-$ counts excitations of qubit s), i.e. $[\hat{H}_{dimer}, \hat{\vartheta}] = 0$, and so $\hat{H}_{dimer}$ is simultaneously diagonalized in an eigenbasis of $\hat{\vartheta}$. Writing the dimer Hamiltonian (1) in such an eigenbasis requires only $4\langle \hat{\vartheta} \rangle$ states. However, as discussed in the main body of paper, we are interested in following the dynamics of coherent states of the cavity fields, which are represented as superpositions over Fock states representing many different photon numbers, and hence such a restriction to fixed polariton number subspaces is insufficient to capture their dynamics. Inclusion of dissipation removes the conservation law, and immediately necessitates use of the full truncated Hilbert space. Additionally, simulating the master equation to capture the dissipative couplings to an environment via quantum trajectory techniques increases the difficulty of the simulation multiplicatively, by the number of stochastic walks to be averaged over (of order hundreds to thousands of individual walks).

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- [1] M. D. Reed, L. DiCarlo, B. R. Johnson, L. Sun, D. I. Schuster, L. Frunzio, and R. J. Schoelkopf., Physical Review Letters **105**, 173601 (2010), ISSN 0031-9007.
 - [2] M. Boissonneault, J. M. Gambetta, and A. Blais, Physical Review Letters **105**, 100504 (2010), ISSN 0031-9007.
 - [3] L. S. Bishop, E. Ginossar, and S. M. Girvin, Physical Review Letters **105**, 100505 (2010), ISSN 0031-9007.

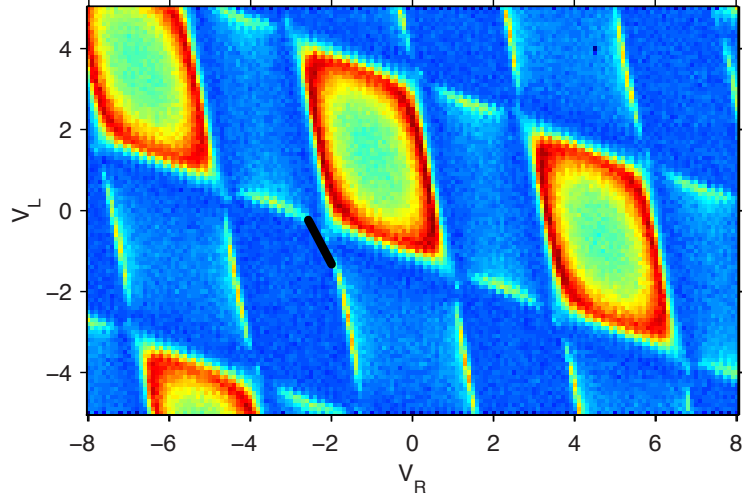


FIG. 1: **S1.** Transmission through an earlier realization of the Jaynes-Cummings dimer with larger dissipation ($\nu_c = 6.182$ GHz, $J = 24$ MHz, $\kappa = 11$ MHz) as a function of V_L and V_R for a fixed frequency ($\nu_d = 6.131$ GHz). The black contour indicates the path in flux space followed in figure (S2), which passes through resonance.

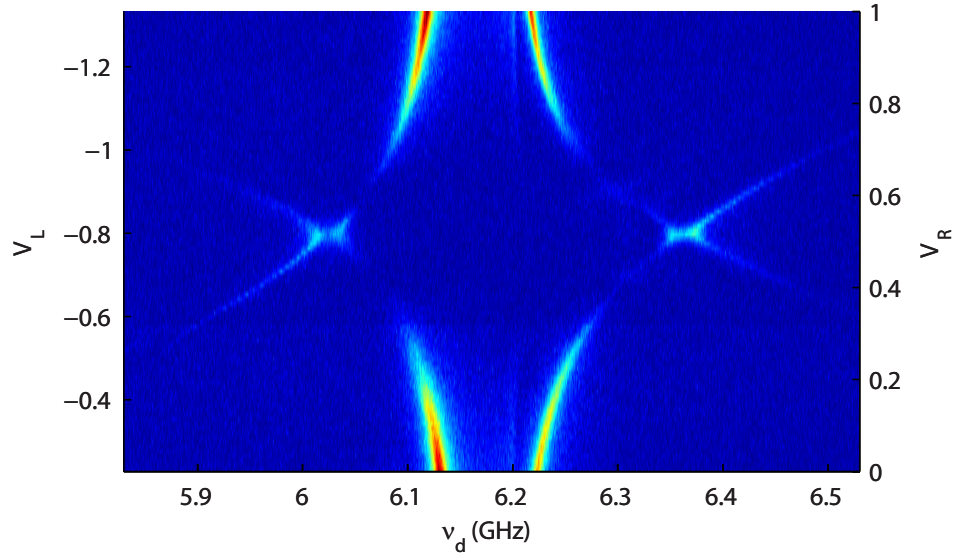


FIG. 2: **S2.** Low power transmission through an earlier realization of the Jaynes-Cummings dimer with larger dissipation, as a function of flux bias and drive frequency. The flux contour was chosen so that both qubits simultaneously enter resonance, one from above ($\nu_a^L > \nu_c$) and the other from below ($\nu_a^R < \nu_c$).

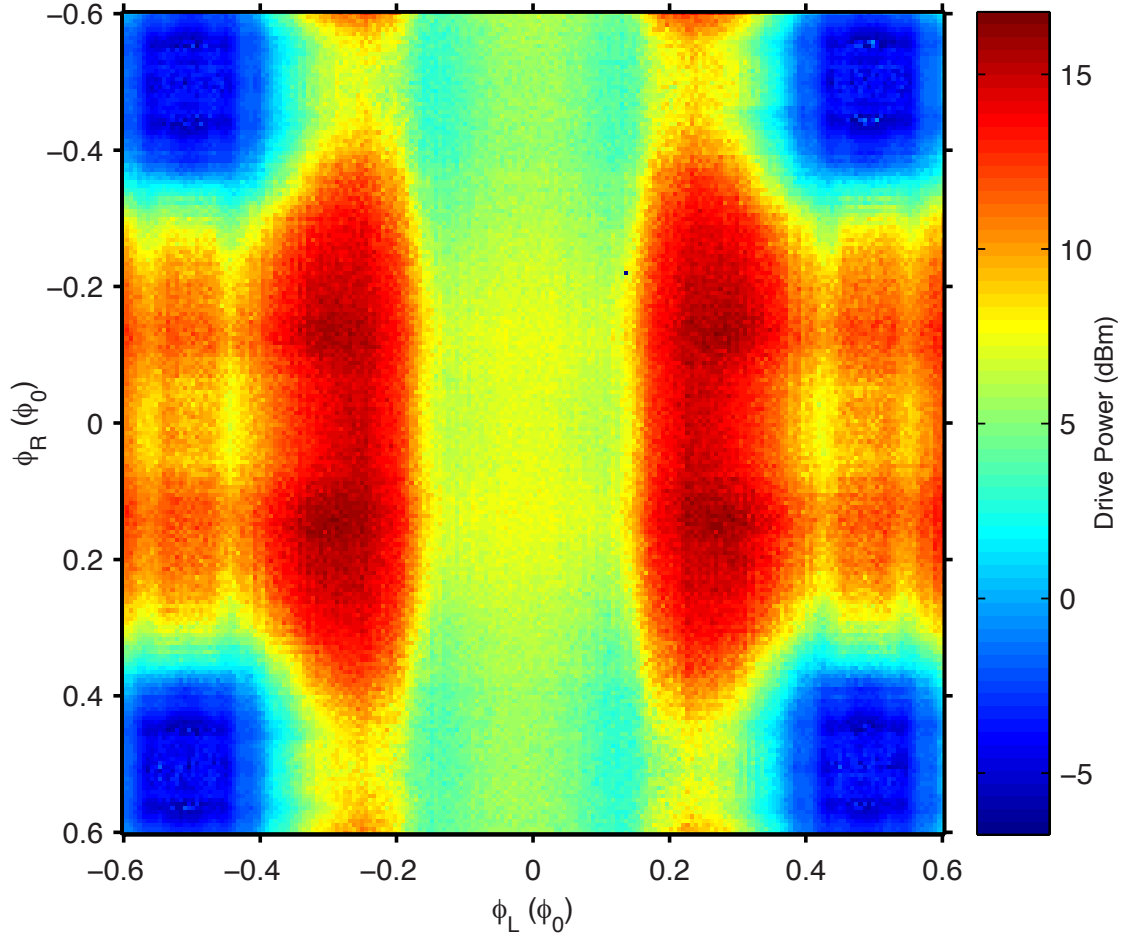


FIG. 3: **S3.** Threshold power for entering the bright state as a function of left and right qubit flux bias for the asymmetric cavity mode. The biases V_L and V_R have been normalized to be expressed in units of the flux quantum Φ_0 and remove cross coupling. At each coordinate in flux space transmission was measured in response to a microwave tone swept in power from -20 dBm to 20 dBm and resonant with the asymmetric cavity mode. Only 0.5 seconds of averaging was needed to obtain a clean signal from which the threshold power was extracted, allowing for rapid calibration of low dissipation systems.

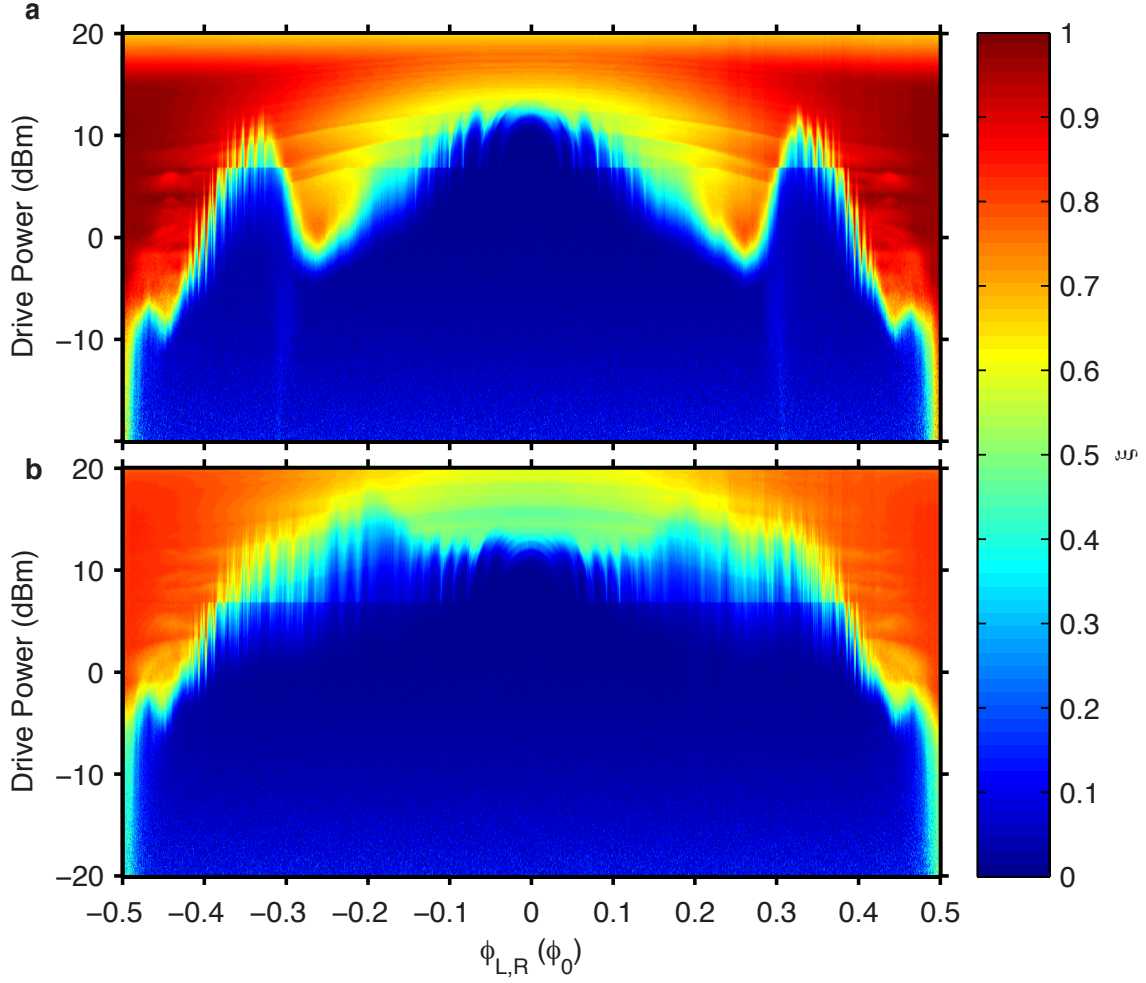


FIG. 4: **S4.** Normalized dimer transmission ξ as a function of drive power along the flux bias contour linearly moving both qubits from $(\Phi_L, \Phi_R) = (-1/2, -1/2)$ to $(\Phi_L, \Phi_R) = (1/2, 1/2)$ for the antisymmetric (a) and symmetric (b) modes. For each bias point power is swept and the transmission is recorded after 20 seconds of averaging with an IF bandwidth of 500 kHz. The long averaging time and high resolution reveal the fine structure of the transition from dark to bright state. The horizontal line visible at 8 dBm is an artifact of the internal source configuration of the generator.

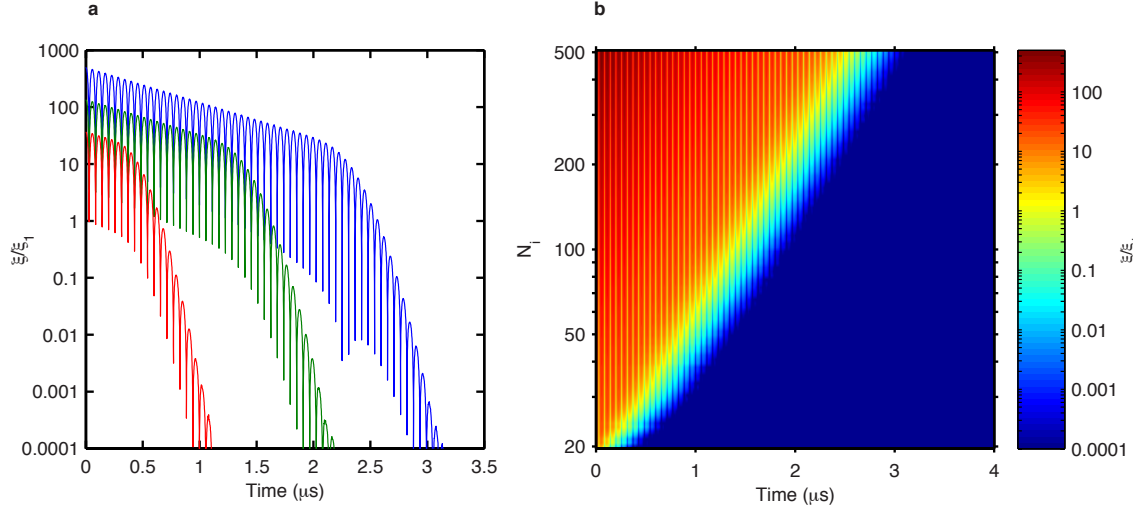


FIG. 5: **S5. a**, Toy model results demonstrating super-exponential decay behavior for $N_i = 500, 136$, and 37 (top to bottom) with decay $\kappa = 225$ kHz and $N_c = 20$. Assuming the existence of (1) a critical photon number N_c , (2) the dynamical behavior of the homodyne signal ξ for the two different phases (linear oscillations for $N > N_c$ and no signal when localized), and (3) Poissonian photon decay, is sufficient to qualitatively reproduce the crossover signature observed in the experiment. **b**, Analog of the phase diagram shown in figure (4c) reproduced using the toy model. Further refinement of the toy model (not shown) by including the effects of critical slowing down as N approaches N_c causes super-exponential decay to occur earlier, but preserves the qualitative structure shown.

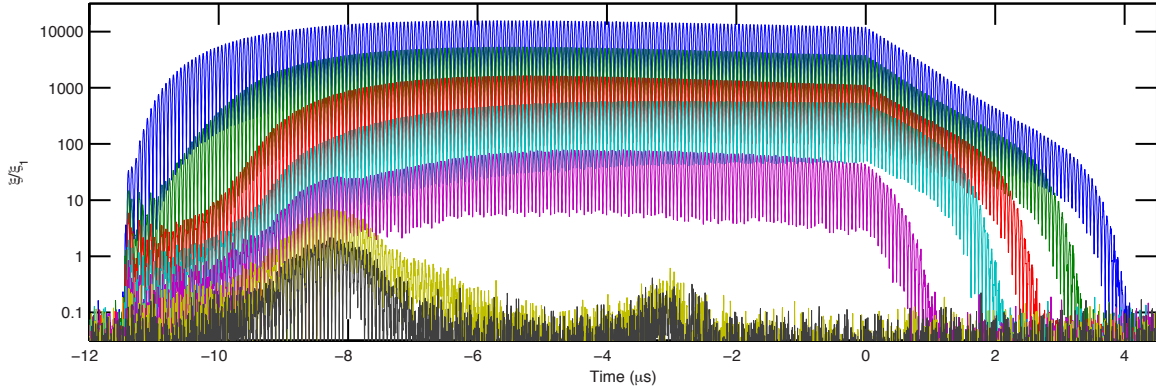


FIG. 6: **S6**. Driven dynamics using a long initialization drive. Here $11.5\mu\text{s}$ ($100/J$) initialization pulses are used with varying drive power (top to bottom 15 dBm to -15 dBm). Drive ends and qubits are tuned into resonance at $t = 0$. The top five traces correspond to those shown in figure (4b) ($N_i \approx 12,000; 3,800; 1,100; 550; 40$). Lower power traces show the effects of finite photon-photon interactions even when the qubits are far detuned during the initialization procedure.