

Supplementary Material

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1 Experimental Details

1.1 Fridge Diagram

The experimental measurement setup is shown in Fig. S1.

1.2 Device Parameters

The qubit was designed, using a finite-element electromagnetic field simulator, to have a charging energy $E_c \sim 65$ MHz. From experiment we determine an upper-bound of ~ 1 GHz on the minimum qubit frequency. A maximum qubit frequency around 6.9 GHz was measured at the onset of the experiment, but due to degradation this shifted to ~ 6.7 and ~ 6.4 GHz on subsequent cool downs. The device was operated near the maximum qubit frequency, for minimum flux noise, for most of the results reported in this paper.

The multimode Purcell effect is the dominant source of qubit decay, and sets the qubit spontaneous decay rate $\gamma/2\pi \sim 1.6$ MHz. In comparison, the qubit lifetimes in standard (~ 7 mm) cavities (fabricated in the same facilities) typically exceed 1-2 μ s.

While box and chip modes were predominantly suppressed by extensive wire-bonding and shielding, we see evidence of spurious modes (such as near 5 and 7 GHz). As our experiments were conducted sufficiently far from these modes, we consider them to have negligible impact on the results presented here.

1.3 Coupling

Multimode strong coupling is accessible using a long superconducting CPW cavity due to the scaling of g_n/FSR with cavity length, L . Since CPW cavities are essentially one-dimensional and the transverse dimension does not support modes in the microwave regime, we only need to consider longitudinal harmonics.

The electric field of a photon in the fundamental mode of a cavity, $E_0 = \sqrt{\frac{\hbar\omega_0}{\epsilon_0 V_{mode}}} \sim 1/L$, as $\omega_0 = FSR \sim 1/L$ and $V_{mode} \sim L$. Thus the field E_m , corresponding to a photon of frequency ω_m , scales as $E_m = E_0 \sqrt{m+1}$. Coupling strength, $g_m = \frac{P_{12}E_m}{\hbar} \cos(k_m x_o) = g_0 \sqrt{m+1}$, where P_{12} is the dipole matrix element and the qubit is fixed at location x_o in our 1D cavity. In these cavities, we are interested in investigating modes of a particular frequency (the maximum qubit frequency), rather than a fixed mode number. Therefore, for a cavity of length L , if this frequency corresponds to the n th harmonic, then $g_n/FSR \sim \sqrt{L}$.

For this experiment, we extract the couplings, g_m , in the multimode Jaynes-Cummings Hamiltonian,

$$H = \frac{\hbar}{2}\omega_a\sigma_z + \sum_m \hbar\omega_m a_m^\dagger a_m + \sum_m \hbar g_m (\sigma^+ a_m + a_m^\dagger \sigma^-) \quad (1)$$

by fitting the avoided crossings (in the lower power transmission scan, see Fig. 2) to the first excitation manifold.

$$H = \begin{pmatrix} \omega_a & g_0 & g_0\sqrt{2} & \dots & g_0\sqrt{n} \\ g_0 & \omega_0 & & & \\ g_0\sqrt{2} & & 2\omega_0 & & \\ \vdots & & & \ddots & \\ g_0\sqrt{n} & & & & n\omega_0 \end{pmatrix}$$

Therefore, in this method the free parameters we vary to determine the fit are g_0 , ω_0 , $\omega_{a,min}$, and $\omega_{a,max}$. Based on the close correspondence between the data and the fit, we determine that the multimode Jaynes-Cummings is a good representation of this system.

1.4 Standard Spectroscopy

Two-tone spectroscopy or dispersive readout is a standard measurement technique which uses the dispersive shift of a mode to precisely determine the frequency of the qubit [1]. In our multimode strong coupling experiment, such spectroscopy shows that the mode frequency is affected not only by the qubit state, but also by populating the modes nearby to the qubit. This is yet another manifestation of mode-mode interactions mediated via the qubit. The magnitude of transmission, a proxy for the dispersive shift, is indicated by the colorbar in Fig. S2. As the qubit frequency approaches a mode, the strength of interaction between that mode and the measurement mode steadily increases.

This interaction is so strong that the measurement mode shifts substantially even when the qubit is over 200 MHz away. Thus the frequency of the measurement mode needed to be recalibrated for each flux voltage in Fig. S2. Furthermore, to track the qubit in this frequency range with sufficient contrast, the designated measurement mode needed to be changed such that it was farther and farther away from the qubit. For each change of measurement mode, there is a horizontal black line in Fig. S2.

1.5 Ultranarrow Linewidth Versus Qubit and Drive Frequencies

We find that ultranarrow fluorescence is very robust to the flux bias voltage of the transmon qubit as shown in Fig. S3, where we display the linewidth as a function of the flux bias voltage applied to the qubit. This observed robustness will be discussed in the following theoretical analysis. Also displayed is

the integrated fluorescence power against the same bias. The power and linewidth are seen to be inversely related. The drive frequency and power here are both fixed to that which results in the minimum linewidth in Fig. 4 of the main text.

2 Theoretical Discussion

2.1 Effective Hamiltonian and Master Equation

In order to better interpret the intriguing dynamical phenomena observed in our experiment, here we provide a simplified model to illustrate key aspects of the physics. Even though the model is far from a complete representation of the experimental system, we argue that qualitatively similar physics exists in the MMSC regime.

This driven dissipative system can be described by the following master equation ($\hbar = 1$ in our units)

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + \frac{\gamma}{2} (2\sigma^- \rho \sigma^+ - \sigma^+ \sigma^- \rho - \rho \sigma^+ \sigma^-) + \sum_m \frac{\kappa_m}{2} (2a_m^\dagger \rho a_m - a_m^\dagger a_m \rho - \rho a_m^\dagger a_m), \quad (2)$$

$$H = H_0 + i\eta (a_r^\dagger - a_r), \quad (3)$$

$$H_0 = \Delta_a \sigma^z + \sum_m \Delta_m a_m^\dagger a_m + \sum_m g_m (a_m^\dagger \sigma^- + \text{h.c.}), \quad (4)$$

with κ_m the photon loss rate for mode m , and γ is the qubit decay rate. The above Hamiltonian is written in the rotating frame of the drive, and the rotating wave approximation (RWA) has been used for the light-matter interaction. $\Delta_m = \omega_m - \omega_d$ is the detuning between the m th mode and the drive. Likewise, $\Delta_a = \omega_a - \omega_d$ is the detuning between the qubit and the drive. RWA should be a good approximation if we focus on high harmonics (~ 70 in the experiment) of this multimode system, as the relevant parameters for RWA, g_m/ω_m , become very small. Further support for using RWA comes from experiment, where the avoided crossings in Fig. 2 are found to be consistent with the multimode Jaynes-Cummings, derived using the RWA. Consequences of the counter-rotating components of the light-matter interaction will be explored in future work.

η is the amplitude of the external drive, and the drive is assumed to be quasi-resonant with the r th mode. Steady state photon numbers of other modes in the uncoupled system ($g_m = 0$) scale as multiples of $(\kappa/FSR)^2$, and hence are suppressed relative to the resonant term. Therefore we are justified in keeping only the resonant drive. Applying an active drive and dissipation dependent unitary displacement to the state of the system, via $\tilde{\rho} = D^\dagger \rho D$, with

$$D = \exp(\xi(a_r^\dagger - a_r)), \quad \xi = \frac{2\eta}{\kappa_r}, \quad (5)$$

transfers the drive in the master equation onto the qubit. The Hamiltonian appearing in the master equation is now $H = H_0 + \Omega(\sigma^+ + \sigma^-)/2$, where $\Omega = 8g_r\eta/\kappa_r$ is the effective Rabi frequency. We could in fact directly implement such a drive in our experiment by driving the qubit via the flux bias line. For the sake of simplicity we start from this form of the Hamiltonian (externally quasi-resonantly driving the qubit) in what follows, and study a restricted two mode system. Furthermore we disregard the mode resonant with the qubit, focusing on the more interesting sideband phenomena.

The master equation for the simplified two mode model is

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + \frac{\gamma}{2} (2\sigma^- \rho \sigma^+ - \sigma^+ \sigma^- \rho - \rho \sigma^+ \sigma^-) + \sum_{m=1,2} \frac{\kappa_m}{2} (2a_m^\dagger \rho a_m - a_m^\dagger a_m \rho - \rho a_m^\dagger a_m), \quad (6)$$

$$H = \Delta_1 a_1^\dagger a_1 + \Delta_2 a_2^\dagger a_2 + \frac{\Omega}{2} (\sigma^+ + \sigma^-) + g_1 (a_1^\dagger \sigma^- + \text{h.c.}) + g_2 (a_2^\dagger \sigma^- + \text{h.c.}). \quad (7)$$

While the single mode case, where qubit dynamics result in the well-studied single-mode version of the Mollow Triplet [2], can be solved analytically for a special choice of parameters ($\Omega = \Delta$) [3], this is in general not possible for a general multimode system. We thus resort to numerical simulation based on the above equation (6) in the following discussion.

To motivate the numerical exploration of the two mode master equation, we first analyze some properties of this Hamiltonian to gain intuition. Introducing a new basis for the qubit by defining $|\tilde{0}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, $|\tilde{1}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, yields the Hamiltonian

$$H = \frac{\Omega}{2}\tilde{\sigma}^z + \sum_{i=1}^2 \Delta_i a_i^\dagger a_i + \frac{g_i}{2} \left[(\tilde{\sigma}^z - \tilde{\sigma}^+ + \tilde{\sigma}^-) a_i^\dagger + \text{h.c.} \right]. \quad (8)$$

Applying an atomic-state-dependent mode displacement operator $U = \prod_{i=1}^2 \exp\left(\frac{g_i}{2\Delta_i}(a_i - a_i^\dagger)\tilde{\sigma}_z\right)$ to remove $\tilde{\sigma}^z$ from the interaction, we have

$$H = \frac{\Omega}{2}\tilde{\sigma}^z + \sum_{i=1}^2 \Delta_i a_i^\dagger a_i + \frac{g_i}{2} \left[(\tilde{\sigma}^+ V_1 V_2 - \tilde{\sigma}^- V_1^\dagger V_2^\dagger)(a_i - a_i^\dagger) \right], \quad (9)$$

where $V_i = \exp\left(\frac{g_i}{\Delta_i}(a_i^\dagger - a_i)\right) = \exp\left(-(g_i/\Delta_i)^2/2\right) \sum_{m,n} (g_i/\Delta_i)^{m+n} \frac{(-1)^n}{m!n!} a_i^{\dagger m} a_i^n$. This form of the Hamiltonian reveals the qubit-mediated multimode multiphoton interactions at all orders in the light-matter interaction strength. Only certain select terms will be secular in this expansion for a given set of qubit and cavity frequencies, valid under the conditions $g_i/\Delta_i \ll 1$ for all i and certain resonance conditions.

We thus arrive at a finite order multimode multiphoton Hamiltonian. For example, let us assume that $\Delta_1 = -\Delta_2 = \Delta$, $g_1 = g_2 = g$, $\kappa_1 = \kappa_2 = \kappa$, and that the qubit is resonant with the drive frequency. In this special case, the resonance conditions are simply $\Omega \approx m\Delta$, for $m \in (1, 2)$. The effective Hamiltonians for the two different Rabi frequencies ($\Omega = \Delta$ and $\Omega = 2\Delta$) become

$$H_{eff}^{(\Omega=\Delta)} = \Delta a_1^\dagger a_1 - \Delta a_2^\dagger a_2 + \frac{\Delta}{2}\tilde{\sigma}^z + \frac{g}{2}[\tilde{\sigma}^+ a_1 - \tilde{\sigma}^+ a_2^\dagger + \text{h.c.}], \quad m = 1 \quad (10)$$

$$H_{eff}^{(\Omega=2\Delta)} = \Delta a_1^\dagger a_1 - \Delta a_2^\dagger a_2 + \Delta\tilde{\sigma}^z + \frac{g^2}{2\Delta}[-\tilde{\sigma}^+ a_1^2 + \tilde{\sigma}^+ a_2^{\dagger 2} + \text{h.c.}]. \quad m = 2 \quad (11)$$

Note that two photon processes in the second Hamiltonian (11) above are suppressed relative to the single photon ones in (10) by g/Δ . Terms like $\tilde{\sigma}^+ a_2^\dagger$ arise because the second mode carries negative frequency in the rotating frame. The second Hamiltonian is the basis for two photon lasing of driven atoms in a cavity [4]. In our case where only a single qubit is present, keeping high-order quantum correlations beyond semiclassics is essential in explaining relevant coherence properties of the fluorescence light. The full master equation could be derived by applying these transformations on the dissipative part, giving rise to complicated multimode multiphoton dissipative terms. We instead jump directly to the numerical results derived from the two mode master equation (6) that explains some of the qualitative features observed in the experiments.

2.2 Multimode Multiphoton Processes

We use the Monte Carlo Wavefunction approach to numerically simulate the dynamics of this driven dissipative multimode system. In our simulation, the size of the Hilbert space needed grows as $2 \prod_m N_m$, with N_m the photon cutoff for mode m . The rapid growth of the dimension of the Hilbert space limits the number of modes that can be incorporated into the simulation. Expectation values of relevant operators are followed and steady state values for the driven system are presented below. To compute two time correlation

functions such as the power spectrum, $\int_{-\infty}^{\infty} e^{-i\omega\tau} \langle a^\dagger(\tau) a(0) \rangle d\tau$, using this method, we have to employ a two step procedure [5], equivalent to the quantum regression theorem, but at the expense of requiring large numbers of walks to produce reliable results.

To explain the lobe structure and multimode fluorescence observed in our experiment, we first analyze two special limits of the two mode model. Resonant enhancement by multiphoton processes in a single mode i (via the interaction $\sigma^+ a_i^m + \text{h.c.}$), while tuning the qubit drive Ω , leads to peaks in the steady state photon number near subharmonics of the Rabi frequency $|\Delta_i| = \Omega/m, m \in \mathbb{Z}$. This is the physical origin of the lobe structure at far detuned modes in our multimode system, as $g/\Delta \ll 1$ is a good approximation for these modes. Fig. S4 presents numerical results for the special two mode case where the two modes are situated symmetrically around the qubit (and the drive which is resonant with it). Fluorescence from these two modes are equally and simultaneously enhanced. Moreover, the two peaks in photon number and qubit polarization σ^+ deviate from the bare resonance conditions $\Omega = m|\Delta|, m = 1, 2$. Most notably, the qubit inversion σ^z can be positive. The results for a single mode model are also shown for comparison. While multiphoton resonances are apparent in both cases, the two mode system reaches much larger photon numbers in the steady state.

Using adiabatic elimination [6] to integrate out the qubit degrees of freedom, we get a simplified equation for the reduced density matrix of cavity modes

$$\frac{d\rho_c}{dt} = -\frac{2}{3\gamma}([V_{10}, [V_{01}, \rho_c]] + [V_{01}[V_{10}, \rho_c]]) + \sum_{m=1}^2 \frac{\kappa_m}{2} (2a_m^\dagger \rho_c a_m - a_m^\dagger a_m \rho_c - \rho_c a_m^\dagger a_m) \quad (12)$$

where $V_{10} = \frac{g}{2}(a_2 - a_1^\dagger)$, and $V_{01} = V_{10}^\dagger$. This is consistent with earlier results [7]. The effective equation enables us to compute expectation values as $\langle n_2 \rangle = \langle n_1 \rangle = g^2/(3\gamma\kappa)$. If we compare these results with that of a strong-coupling single mode problem [3] where $\langle n \rangle \approx \gamma/(8\kappa)$, we immediately see that qubit dissipation plays a very different role in determining steady state properties of the system. Intuitively, in the single mode case, a bad qubit (or large γ) is needed to create population inversion between "dressed" qubit states $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$, which in turn produces photon emission into the mode $(\tilde{\sigma}^- a_1^\dagger + \tilde{\sigma}^+ a_1)$. However, in the two mode case, population inversion is no longer important. The symmetric two mode configuration ensures that both qubit state flips ($\tilde{\sigma}^-$ and $\tilde{\sigma}^+$) can create photons into the modes $(\tilde{\sigma}^- a_2^\dagger + \tilde{\sigma}^+ a_1^\dagger)$. Consequently, the multimode system is more efficient in harvesting emission from the qubit. These results are confirmed by numerical simulations (see Fig. S5). Additionally, we find through numerical simulation that qubit-mode coupling also plays a different role in determining steady-state photon number in these two systems (see Fig. S6).

The difference also appears in the spectral linewidth (see Fig. S8). While the single mode fluorescence shows no spectral narrowing, the spectral linewidth in the two mode model can be much smaller than the natural linewidth. Beside the simple two photon interaction in a single mode $(\sigma^- a_i^\dagger a_i^\dagger)$ analyzed above, the interaction vertex coupling two different modes $(\sigma^- a_i a_j^\dagger, i \neq j)$ will result in resonant enhancement in modes near harmonics of the Rabi frequency $|\Delta| = m\Omega$. This is demonstrated in Fig. S7 where another special two mode case ($\Delta_2 = 2\Delta_1$) is investigated. Similarly to the previous case, this effect leads to co-enhancement of fluorescence at two modes, as evidenced by the brightening of the second mode at $\Delta \sim 80$ MHz. As this is a second-order effect induced by the first mode, N_2 is much smaller than N_1 . For stronger coupling, we would expect the difference to be smaller.

In our strong coupling multimode system, all multimode multiphoton terms have to be taken into account, which makes the analysis very cumbersome. Clearly a different theoretical approach is in order here. The competition between these processes will generally make resonances broader and less prominent. The numerical simulations qualitatively agree with our experimental results.

2.3 Multimode Spectral Narrowing

Ultranarrow resonance fluorescence has been predicted for the single mode cavity [8], where coherence between dressed states of the driven system arises from coupling to common vacuum fluctuations. The resultant spectrum linewidth of the mode could be much smaller than the natural linewidth of the bare system. In fact, the linewidth of the sideband is proportional to the inverse of the steady state photon number $1/\langle n \rangle$. We provide numerical evidence that this is also true in the multimode system. It is worth noting that reference [8] only considered a special value of the parameters, where $\Omega = \Delta$ and $g \sim \gamma$. Here we generalize their result to explain our observations.

In Fig. S8, for the symmetric two mode system, it is shown that spectral narrowing appears for a range of the Rabi frequency Ω . Moreover, the linewidth is inversely proportional to the photon number (see also Fig. S4(a)) such that the linewidth reaches its minimum as the photon number reaches its maximum at $\Omega = 106$ MHz. The left mode also shows the same amount of spectral narrowing (data not shown). The spectral narrowing in this simplified model doesn't reach that of our experiments (a factor of ~ 3.3 in the simulation vs. ~ 17 in the experiment), yet it displays qualitatively similar features. Moreover, the relation $\kappa_{fluor} \sim 1/\langle n \rangle$ suggests that the linewidth could be made much narrower than that of the current simplified model, when reaching larger steady state photon numbers. For the interacting multimode system, the proliferation of the dressed states in the high excitation manifolds leads to enhanced generation of coherence among the dressed states from a collective interference, which relies on the near resonance of many dressed state level spacings. Concomitant with the collective suppression of dissipation for the steady state then is the rise in the steady state occupation. Simulations of such a multimode system demands more computational resources due to the growth of the Hilbert space. More extensive results will be presented elsewhere.

Spectral narrowing is generally very insensitive to the qubit frequency. For finite drive-qubit detuning Δ_a , the effective Rabi frequency is $\Omega = (\Omega_0^2 + \Delta_a^2)^{1/2}$, with Ω_0 the amplitude of the drive. Based on the above analysis, spectral narrowing occurs when Ω is within the range $\Delta \pm \delta(\Omega_0)$, where Δ is again the mode frequency measured with respect to the drive (i.e. its detuning), and $\delta(\Omega_0)$ is generally determined by the number of modes, the coupling strength and the dissipation. For relatively small detuning Δ_a and large Rabi frequency Ω_0 , this gives a large effective range of qubit frequency $\delta(\Delta_a) \sim \frac{\Omega}{\Delta_a} \delta(\Omega_0)$ over which narrowing occurs. It is worth noting that even if the drive is not resonant with the qubit and the two modes are not symmetric around the drive, spectral narrowing still exists in our simulation.

For a single mode system, it was argued that spectral narrowing requires that the qubit decay rate γ be comparable to the coupling strength g , i.e. the moderate coupling regime [8]. We therefore study the effect of qubit decay rate γ on steady state properties of the two mode system. Fig. S9(a) presents photon number N_1 versus Rabi frequency for a different γ value. From this result, we conclude that spectral narrowing survives when going from moderate coupling to the strong coupling regime, which we attribute to the fact that the eigenstates of the multimode system in high excitation manifolds are more densely spaced than in the single mode system. Dynamically generated coherence between these closely spaced states then results in spectral narrowing of many modes simultaneously.

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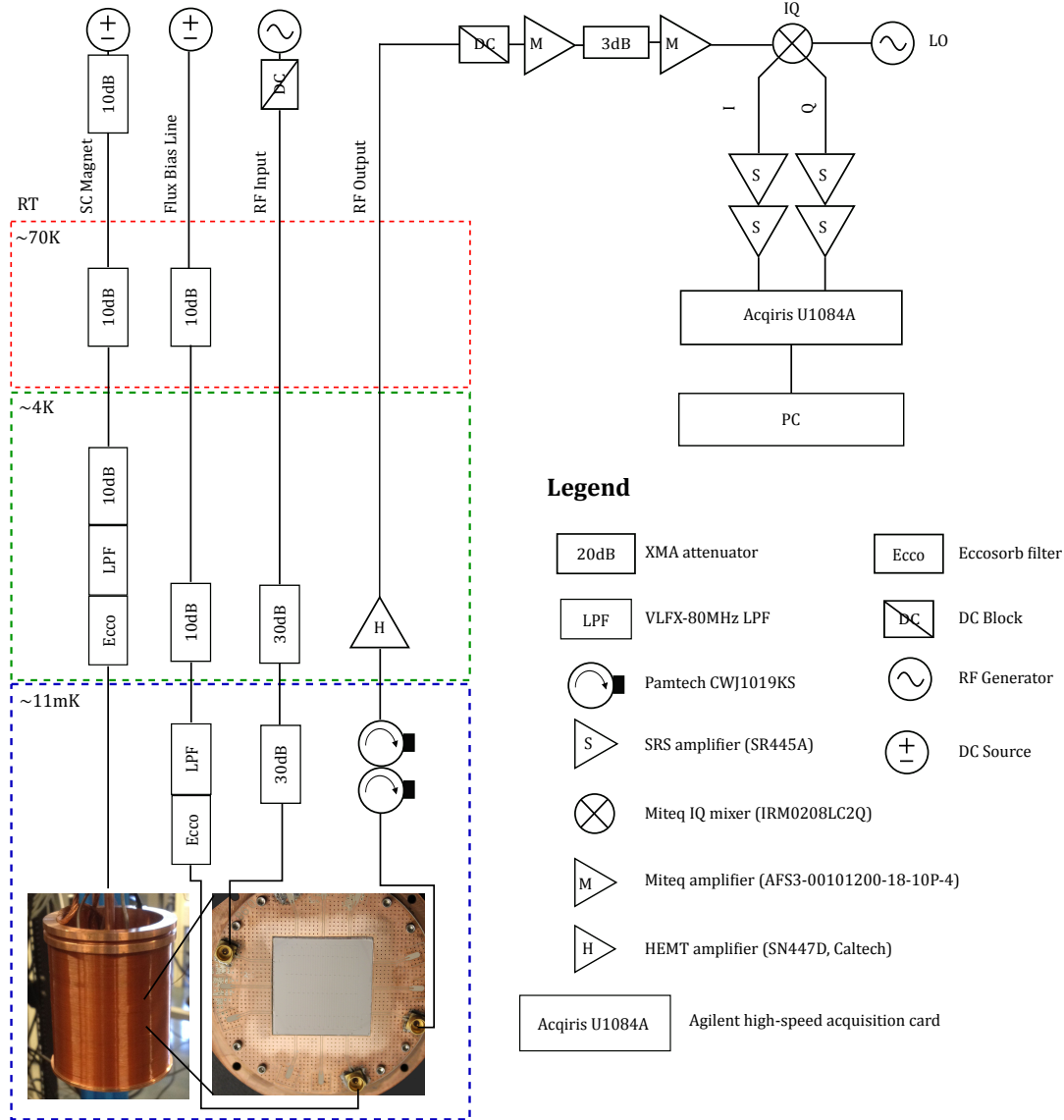


Figure S1: Schematic representation of measurement setup. Device is mounted within the superconducting magnet, inside a dilution refrigerator. Experimental results presented in the paper were obtained from this general setup, with varied digital processing.

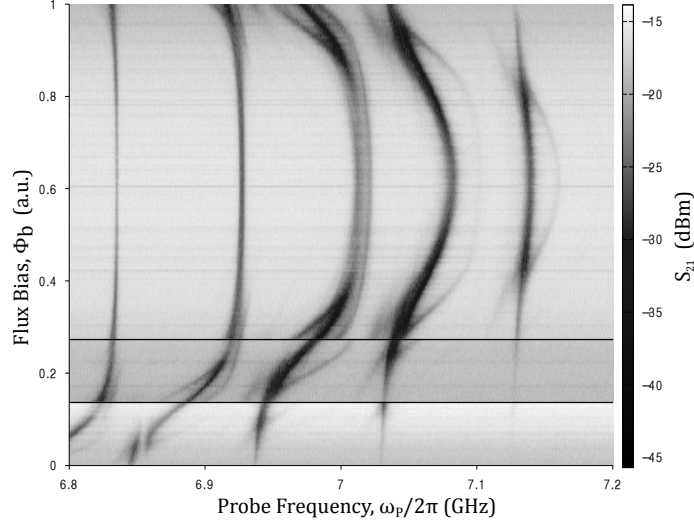


Figure S2: Mode-mode interaction in dispersive readout. Tracking the change in dispersive shift for measurement mode, at ω_M , with varying probe frequency, ω_p , shows the impact of populating nearby modes. The magnitude of the dispersive shift is related to the detuning of a mode from the qubit (at 7.08GHz for $\phi_b = 0.6$). The qubit frequency is tuned using the on-chip flux bias line. Fainter curves are attributed to higher order processes. Horizontal black lines indicate a change of ω_M (6.469, 6.653, and 6.747 GHz from top to bottom), chosen to maintain sufficient distance from the qubit.

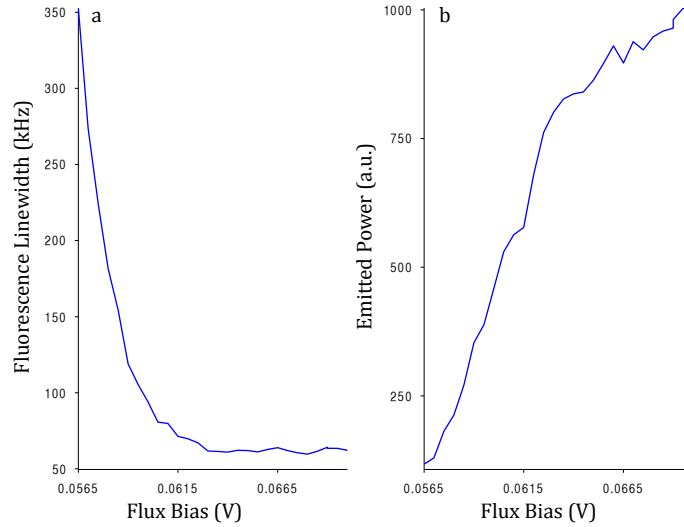


Figure S3: Measured linewidth and emitted power versus flux bias. Spectral narrowing appears over a wide range of flux bias, corresponding to over 150 MHz of the qubit frequency. The other parameters are the same as in Fig. 4 in the main text. The fluorescence power and the linewidth are inversely related.

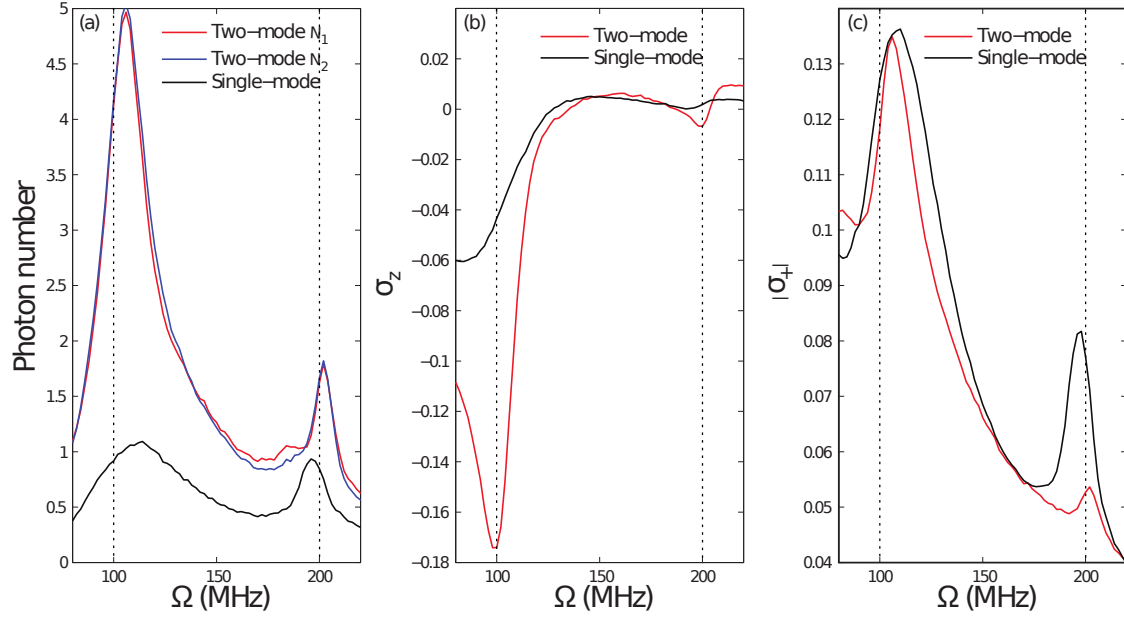


Figure S4: Steady state photon number, qubit inversion and polarization, first parameter set. Parameters of the two mode system are $\Delta_1/2\pi = -\Delta_2/2\pi = 100$ MHz, $\kappa_1/2\pi = \kappa_2/2\pi = 1$ MHz, $\gamma/2\pi = 15$ MHz, $g_1/2\pi = g_2/2\pi = 15$ MHz. Parameters of the single mode system are $\Delta/2\pi = 100$ MHz, $\kappa/2\pi = 1$ MHz, $\gamma/2\pi = 15$ MHz, $g/2\pi = 15$ MHz. These plots include 24 different Rabi frequencies Ω with 1,000 quantum walks per Rabi frequency in the numerical simulations.

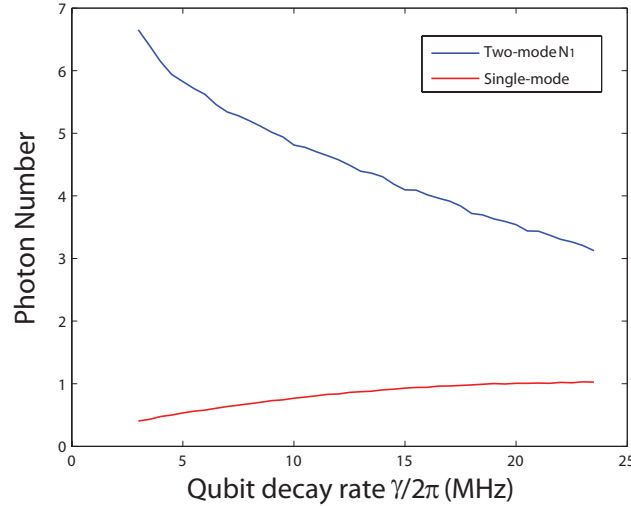


Figure S5: Comparison of photon number dependence on qubit decay for the right mode, in the symmetric two mode and single mode systems. Parameters of the two mode system are $\Delta_1/2\pi = -\Delta_2/2\pi = 100$ MHz, $\kappa_1/2\pi = \kappa_2/2\pi = 1$ MHz, $g_1/2\pi = g_2/2\pi = 15$ MHz, $\Omega/2\pi = 100$ MHz. For the single mode system, $\Delta_1/2\pi = 100$ MHz, $\kappa_1/2\pi = 1$ MHz, $g_1/2\pi = 15$ MHz, $\Omega/2\pi = 100$ MHz. The symmetric two mode configuration yields higher photon numbers that decrease with increasing qubit decay, unlike the single mode case.

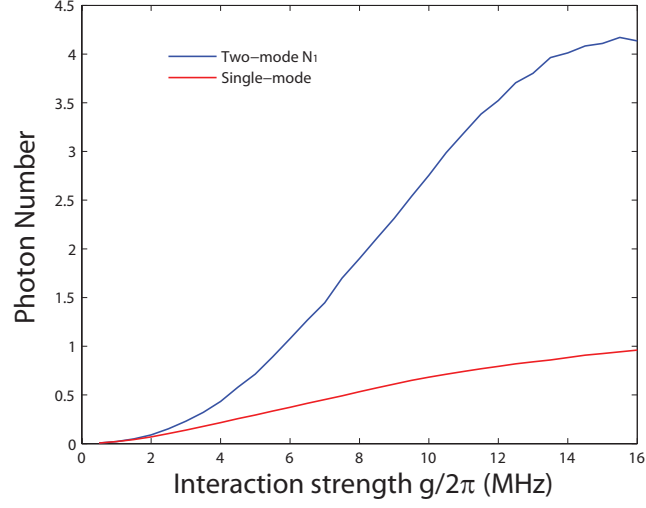


Figure S 6: Comparison of photon number dependence on interaction strength for the right mode, in the symmetric two mode and single mode systems. Parameters of the two mode system are $\Delta_1/2\pi = -\Delta_2/2\pi = 100$ MHz, $\kappa_1/2\pi = \kappa_2/2\pi = 1$ MHz, $\gamma/2\pi = 15$ MHz, $\Omega/2\pi = 100$ MHz. For the single mode system, $\Delta_1/2\pi = 100$ MHz, $\kappa_1/2\pi = 1$ MHz, $\gamma/2\pi = 15$ MHz, $\Omega/2\pi = 100$ MHz. The two mode system reveals a distinct enhancement of photon number, with increasing interaction strength, over the single mode case [9].

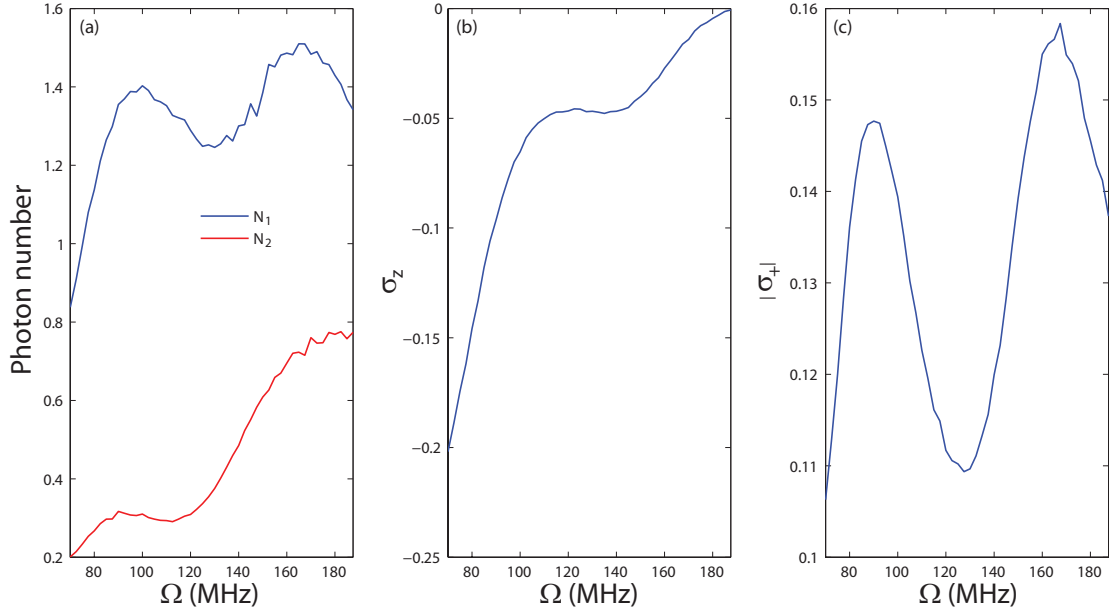


Figure S 7: Steady state photon number, qubit inversion and polarization, second parameter set. Parameters of the two mode system are $2\Delta_1/2\pi = \Delta_2/2\pi = 160$ MHz, $\kappa_1/2\pi = \kappa_2/2\pi = 1$ MHz, $\gamma/2\pi = 15$ MHz, $g_1/2\pi = g_2/2\pi = 25$ MHz. These plots include 24 different Rabi frequencies Ω with 1,000 quantum walks per Rabi frequency in the numerical simulations.

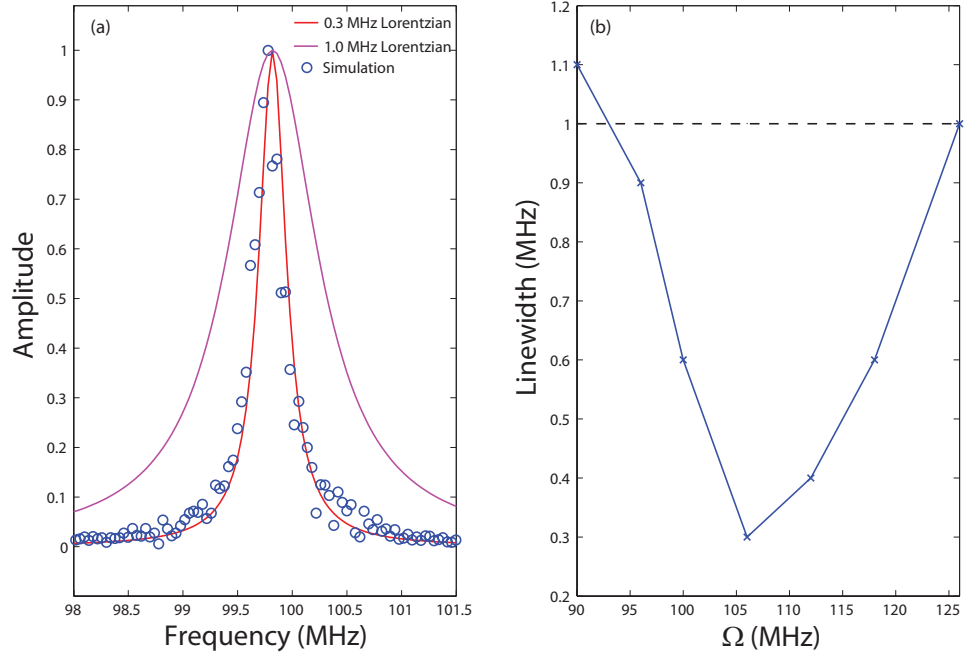


Figure S8: Fluorescence spectrum of the right mode in a two mode system. (a) Parameters of the simulation are the same as Fig. S4, with $\Omega/2\pi = 106$ MHz, where the photon number reaches its peak. This figure includes 12,000 quantum walks in the numerical simulation. The spectrum is generated by Fourier transforming the two time correlation function of the mode operator. The fluorescence linewidth is more than 3 times narrower than the natural mode linewidth. (b) Fluorescence linewidth versus Rabi frequency. The black dashed line indicates the natural linewidth of the mode (1 MHz).

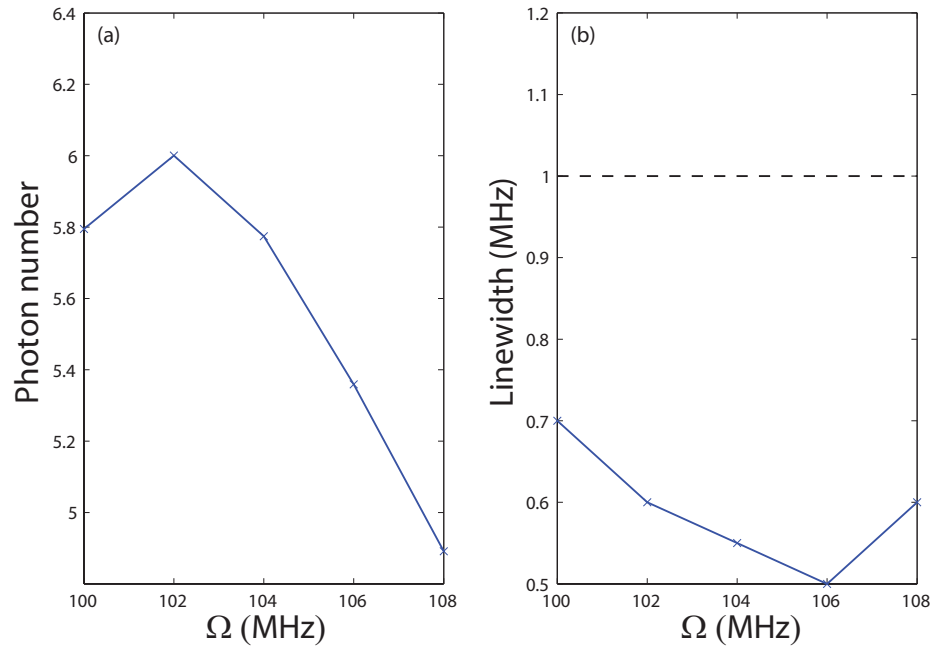


Figure S9: Photon number and linewidth of the right mode for varying drive amplitude. Parameters are the same as Fig. S4 except that $\gamma = 5$ MHz instead of 15 MHz. The sharpest line does not correspond to the brightest fluorescence. Nevertheless, it is a factor of 2 narrower than the natural linewidth of the bare system. The black dashed line indicates the natural linewidth of the mode (1 MHz).