

Enhancement of laser power-efficiency by control of spatial hole burning interactions

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The laser is an out-of-equilibrium nonlinear wave system where the interplay of the cavity geometry and nonlinear wave interactions mediated by the gain medium determines the self-organized oscillation frequencies and the associated spatial field patterns. In the steady state, a constant energy flux flows through the laser from the pump to the far field, with the ratio of the total output power to the input power determining the power-efficiency. Although nonlinear wave interactions have been modelled and well understood since the early days of laser theory, their impact on the power-efficiency of a laser system is poorly understood. Here, we show that spatial hole burning interactions generally decrease the power-efficiency. We then demonstrate how spatial hole burning interactions can be controlled by a spatially tailored pump profile, thereby boosting the power-efficiency, in some cases by orders of magnitude.

The power-efficiency of lasers is a key property to be maximized to reduce the energy requirements for on-chip applications, to facilitate thermal management and to pack lasers into smaller volumes. Earlier work on solid-state lasers addressed the quantum design of the gain medium and optimization of carrier transport, thereby demonstrating strong improvements in power-efficiency^{1,2}. Far less systematic work has addressed the fundamental factors limiting the power-efficiency of lasers and effective methods to overcome these, taking into account specific resonator properties. In particular, the impact of spatial hole burning interactions is poorly understood³. A case in point are the findings of ref. 4, where the output power of microcavity quantum cascade lasers was found to increase exponentially with boundary deformation, resulting in a power enhancement of over two orders of magnitude with respect to identical lasers of circular cross-section. After more than a decade of theoretical development, the fundamental factors behind this dramatic increase in power-efficiency remain unknown. The goal of this article is to provide a theoretical analysis of the mechanisms that enable such dramatic improvements in laser power-efficiency.

Typically, the pump power in a microlaser is deposited uniformly across the entire cavity area (Fig. 1a) and lasing naturally occurs in cavity modes with the longest cavity lifetimes. Here, we show the existence of a maximally power-efficient ‘optimally outcoupled mode’ that may never turn on in a uniformly pumped cavity due to spatial hole burning interactions. We further show that a non-uniform pump distribution designed to selectively pump the optimally outcoupled mode can lead to strong power enhancements.

Spatially non-uniform pump distributions can be realized by fabricating patterned contacts⁵, by spatially non-uniform doping in the case of current injection lasers^{6,7}, or by using spatial light modulators or special lenses for optically pumped lasers^{8,9} (Fig. 1b). We discuss the optimal spatial profile of the pump for a given resonator to maximize the power-efficiency, assuming a fixed internal quantum efficiency for the gain medium. We demonstrate a proof-of-principle for a whispering gallery resonator where a remarkable power enhancement factor of 185 can be achieved. The power enhancements we quote in the following are the relative power enhancements of the total multimode laser power (summed

over all modes) for the selective pumping (Fig. 1b) versus the uniform pumping (Fig. 1a) of the identical resonator with the same gain volume.

Multimode lasing in open systems

Our analysis is based on a recent reformulation of the semiclassical laser equations—the steady-state *ab initio* laser theory (SALT)^{10–12}—which allows us to describe lasing characteristics that are independent of the specific details of the gain medium (Supplementary Section 1).

SALT finds the steady-state solution of the semiclassical laser equations in the general multi-periodic form

$$E^+(\mathbf{r}, t) = \sum_{\mu=1}^N \Psi_{\mu}(\mathbf{r}) e^{-i\Omega_{\mu}t}, \quad P^+(\mathbf{r}, t) = \sum_{\mu=1}^N p_{\mu}(\mathbf{r}) e^{-i\Omega_{\mu}t}$$

where N is the number of lasing modes at a given pump power and $E^+(\mathbf{r}, t)$ and $P^+(\mathbf{r}, t)$ are the positive frequency components of the electric field and polarization density. The nonlinear lasing modes Ψ_{μ} and laser frequencies Ω_{μ} can be obtained from the following set of coupled Helmholtz equations for a homogeneously broadened gain medium:

$$\left[\nabla^2 + (\epsilon_c(\mathbf{r}) + \epsilon_g(\mathbf{r}))\Omega_{\mu}^2 \right] \Psi_{\mu}(\mathbf{r}) = 0, \quad \mathbf{r} \in \text{cavity} \quad (1)$$

with an effective dielectric function consisting of the passive contribution $\epsilon_c(\mathbf{r}) = n_c^2(\mathbf{r})$ and an active contribution $\epsilon_g(\mathbf{r})$ from the gain medium

$$\epsilon_g(\mathbf{r}) = \frac{\gamma_{\perp}}{\Omega_{\mu} - \omega_a + i\gamma_{\perp}} \frac{D_0(\mathbf{r})}{1 + \sum_{\nu=1}^N \Gamma_{\nu} |\Psi_{\nu}(\mathbf{r})|^2} \quad (2)$$

These equations are to be interpreted as a set of nonlinear eigenvalue equations to be solved for the lasing frequencies Ω_{μ} and the nonlinear lasing modes Ψ_{μ} for a given pump distribution $D_0(\mathbf{r})$ capturing directly the steady-state behaviour. The complex linear index of refraction $n_c(\mathbf{r})$ defines the cavity and $\epsilon_g(\mathbf{r})$ contains the nonlinear spatial hole burning interactions exactly, to infinite

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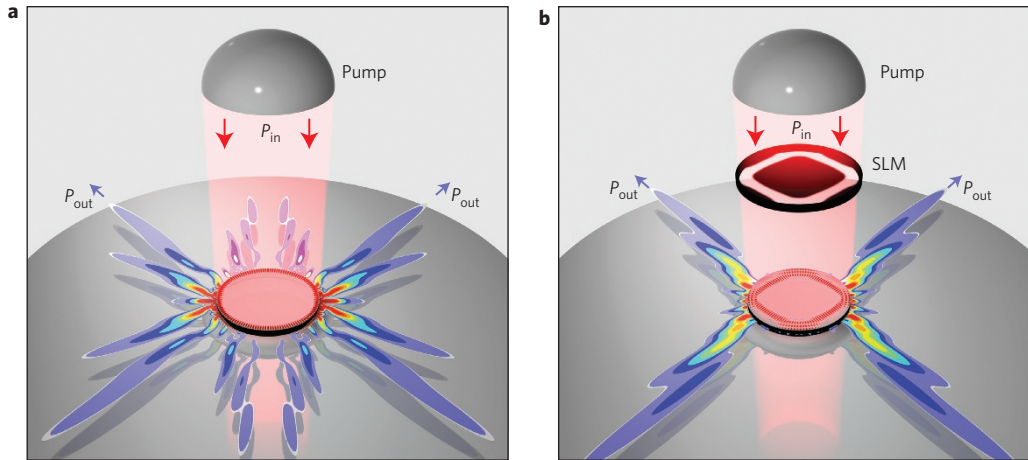


Figure 1 | Comparison of different optical pumping schemes and their corresponding emission patterns. A high-Q whispering-gallery mode is excited by a uniform pump **(a)** and a spatial light modulator (SLM) is used to shape the beam and select a diamond-shaped mode **(b)**. Selective pumping can also be achieved by a special lens⁹ that focuses the total incoming power to a reduced total area or by patterned contacts in the case of current-injection lasers. Both mechanisms are discussed in the main text.

order in the intensity of the lasing modes. We have taken the speed of light in vacuum $c = 1$, ω_a is the atomic transition frequency, $\gamma_{\parallel}$ and $\gamma_{\perp}$ are the inversion and polarization relaxation rates, g is the transition dipole matrix element between the lasing levels, and $\Gamma_{\mu} \equiv \gamma_{\perp}^2 / [\gamma_{\perp}^2 + (\Omega_{\mu} - \omega_a)^2]$ is the Lorentzian gain curve evaluated at the frequency of the lasing mode μ . The field $\Psi_{\mu}(\mathbf{r})$ is dimensionless and measured in units of $E_0 = \hbar \sqrt{\gamma_{\parallel} \gamma_{\perp}} / 2g$. The pump is given by $D_0(\mathbf{r}) = D_0 f(\mathbf{r})$, where D_0 measures the total power injected in terms of the spatially averaged inversion density, and $f(\mathbf{r})$ is normalized in a way to maintain the correspondence between uniformly and selectively pumped cavities (Supplementary Section 1.1).

Equations (1) describe the radiation field both inside and outside the cavity and their direct numerical solution on a grid generally scales with the size of the entire scattering domain. The original semiclassical laser equations (Supplementary equations 1–3) as well as the steady-state equations (1) can, however, also be solved by projecting them onto a suitable linear basis that scales only with the size of the cavity. This is what is done here, accounting for openness exactly by expanding each lasing mode $\Psi_{\mu}(\mathbf{r})$ further into a non-Hermitian basis set $\{\phi_m(\mathbf{r})\}$, the constant-flux (CF) basis¹⁰:

$$\Psi_{\mu}(\mathbf{r}) = \sum_m a_m^{\mu} \phi_m(\mathbf{r}; \Omega_{\mu}) \quad (3)$$

This basis, as well as the associated complex-valued eigenfrequencies $\omega_{\mu} = \nu_{\mu} - i\kappa_{\mu}$, are parametrically dependent on Ω_{μ} . We note that the CF basis is a complete set of states to expand an arbitrary field that oscillates at frequency Ω_{μ} inside an open cavity and is different from quasi-bound modes (poles of the scattering matrix)¹³, because they carry a constant flux to the far field. In the following sections we further develop SALT projected onto the CF basis, to determine and excite the optimally outcoupled mode.

Optimally outcoupled mode

The optimally outcoupled mode of a given resonator is the mode that provides the largest slope efficiency for the total optical power in the far field. The far-field power radiated by a lasing mode μ at a given pump power D_0 can be expressed as $P_{\text{out}} = C_{\mu} |a_{\mu}(D_0)|^2$, given that the lasing mode can be approximated by a single CF state, typically an excellent approximation for high-Q cavities (Supplementary Section 1.1). We call C_{μ} the ‘outcoupling efficiency’ for mode μ and find by calculation that it is inversely proportional to the ‘outcoupling Q-factor’ $Q_{\mu} \equiv \nu_{\mu} / 2\kappa_{\mu}$

(Supplementary Section 1.2 and Supplementary Fig. 1), defined using the complex-valued resonances $\omega_{\mu} = \nu_{\mu} - i\kappa_{\mu}$ in the absence of absorption.

For a microcavity of arbitrary shape and uniform index of refraction $n_c = \sqrt{\epsilon_c} \equiv n_0 + in_1$ ($n_0 \gg n_1 > 0$), the threshold of mode μ is given by

$$D_{\text{th}}^{\mu} \approx n_0^2 (Q_{\mu}^{-1} + Q_{\text{abs}}^{-1}) \equiv D_{\text{out}}^{\mu} + D_{\text{abs}} \quad (4)$$

where $Q_{\text{abs}} \equiv n_0 / 2n_1$ is the absorptive Q-factor (Fig. 2a) neglecting spatial hole burning interactions.

Now considering P_{out} and D_{th}^{μ} together, we note that while a large Q_{μ} implies a low threshold (and hence a large $|a_{\mu}|^2$), it also gives rise to a low outcoupling efficiency C_{μ} . The total output power P_{out} is maximized at an optimal value of Q_{μ} , the optimal Q-factor (Supplementary Section 1.2):

$$Q_{\text{opt}} = \frac{Q_{\text{abs}}}{\sqrt{\beta} - 1} \quad (5)$$

where $\beta \equiv D_0 / D_{\text{abs}}$ is the pump strength in terms of the minimum threshold given by equation (4). Equation (5) shows that the optimal Q-factor is close to the absorptive Q-value. This rule of thumb is well-known and can be rigorously derived for a Fabry–Perot cavity^{14–16}. Similar outcoupling criteria in the absence of self-saturation have also been used for fibre-coupled microcavity lasers^{17–19}. Here, we present the rigorous foundations for this rule and generalize this condition to an arbitrarily complex open cavity, taking into account self-saturation effects—the latter manifested in the pump dependence (β) of the optimal outcoupling condition (Fig. 2b).

Mode discrimination by spatially selective pumping

Microlasers typically operate in the multimode regime where spatial hole burning interactions with lower-threshold modes can entirely suppress the optimally outcoupled mode, identified above in the absence of these interactions. Here, we discuss how the optimally outcoupled mode can be favoured to lase, and the rest of the modes are suppressed by spatially tailoring the pump profile.

To this end, we resort to finding an ideal spatial pump profile that overlaps strongly with the optimally outcoupled mode. Spatially non-uniform pumping schemes have previously been used to control various lasing characteristics, including emission directionality, thresholds and frequencies, in microdisks⁸, spirals^{9,20}, asymmetric resonant cavities (ARC)^{21,22} and random lasers^{23–28},

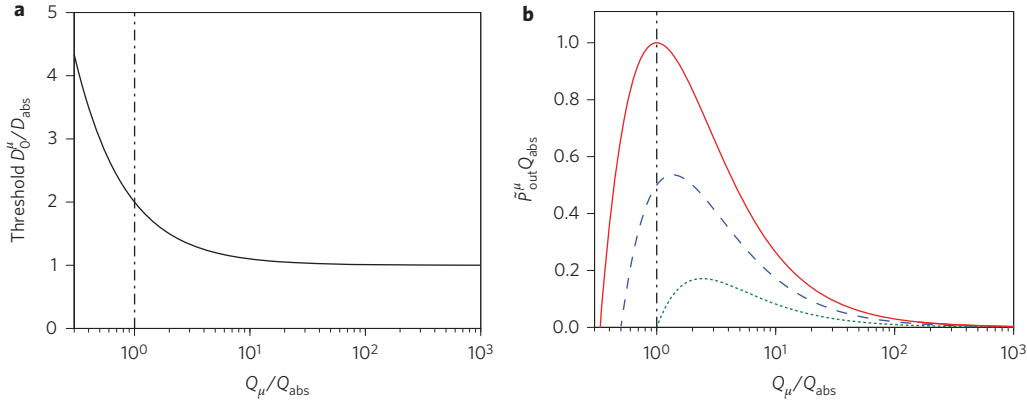


Figure 2 | Characteristics of the optimally outcoupled mode. The threshold (a) is low while the output intensity (b) peaks. In a, thresholds as a function of the outcoupling Q-factor for uniform pumping (modal interactions neglected) are shown. The vertical dash-dotted line marks the absorptive Q-value. In b, normalized output intensity curves are shown for three values of pump strength: $\beta = 2$ (dotted), 3 (dashed) and 4 (solid). $\tilde{P}_{\text{out}}^{\mu}$ is the output power given by Supplementary equation (35). For this calculation χ_{μ} and Γ_{μ} are taken to be 1.

mostly in the single-mode regime and generally in a rather ad hoc manner. To our knowledge, the present work is the first to propose it as a scheme to systematically boost laser power in the multimode regime.

Consider a normalized spatial pump profile $f(\mathbf{r})$ in the plane of the microcavity. Supplementary Section 1.3 shows that the threshold of mode μ , in terms of the total pump power delivered to the entire cavity area, is modified from D_{th}^{μ} to approximately $D_{\text{th}}^{\mu}/\text{Re}[f_{\mu}]$. Here, the complex-valued ‘pump overlap factor’ is defined by

$$f_{\mu} \equiv \int_{\text{cavity}} f(\mathbf{r}) \phi_{\mu}^2(\mathbf{r}) d\mathbf{r} \quad (6)$$

Once the optimally outcoupled mode has been identified for a given cavity, we introduce a spatial pump profile that leads to a large overlap factor for this target mode, giving rise to a pump-focusing effect that reduces its threshold. For the higher-Q modes that have lower thresholds with a uniform pump profile, the overlap factors can in principle be made smaller than unity, leading to increased thresholds and inhibiting unwanted modes from lasing.

Case study: the whispering gallery microlaser

For a proof-of-principle demonstration of our approach we selectively pump a microdisk laser, treated as a two-dimensional system using the effective index approach. This is a case where analytical formulae can be obtained for many of the factors discussed in the previous sections, including the full nonlinear steady-state solutions of the semiclassical laser equations expressed in equation (1) (Supplementary Section 1.1).

Microdisk lasers utilize total internal reflection to trap light for a long period of time, and the field distribution of each high-Q whispering-gallery mode changes little with nonlinearity. In this case, a lasing mode can be represented by a single CF state, characterized by its angular and radial quantum numbers $\mu = (m, \eta)$:

$$\phi_{\mu}(r, \phi) = \begin{cases} \frac{1}{\zeta_{\mu}} J_m(n_c \omega_{\mu} r) e^{im\phi}, & r < R \\ \frac{1}{\zeta_{\mu}} \frac{J_m(n_c \omega_{\mu} R)}{H_m^+(\Omega_{\mu} R)} H_m^+(\Omega_{\mu} r) e^{im\phi}, & r > R \end{cases} \quad (7)$$

where J_m, H_m^+ are Bessel and Hankel functions of the first kind and $\zeta_{\mu} = [2\pi \int_{\text{cavity}} J_m^2(n_c \omega_{\mu} r) r dr]^{1/2}$ is a normalization factor. The real-valued lasing frequency Ω_{μ} and the complex-valued CF frequency ω_{μ} are related by a generalized frequency pulling expression (Supplementary equation (24)). The exact expression for the

outcoupling efficiency (as defined in the previous section) associated with each mode is

$$C_{\mu} = G \frac{\omega_a}{\Omega_{\mu}} \left| \frac{1}{\zeta_{\mu}} \frac{J_m(n_c \omega_{\mu} R)}{H_m^+(\Omega_{\mu} R)} \right|^2 \quad (8)$$

where G is a geometry-dependent factor defined in Supplementary equation (34).

Under uniform pumping, the threshold of mode μ in single-mode operation reduces exponentially as the incidence angle $\chi_{\mu} \approx \sin^{-1}(m/n_0 \Omega_{\mu} R)$ becomes larger than the critical angle $\chi_c = \sin^{-1}(1/n_0)$ (Fig. 3a), until it reaches the minimum threshold D_{abs} . The small fluctuation of the thresholds near the asymptote is due to the finite width of the gain; modes closer to the gain centre ω_a (Fig. 3b) have slightly lower thresholds, which is not considered in the approximate expression shown in equation (4).

We take the cavity refractive index $n_c = 3.3 + 10^{-4}i$, which leads to an absorptive Q-factor $Q_{\text{abs}} = 1.65 \times 10^4$. Because the outcoupling Q-factors can be many orders of magnitude larger than Q_{abs} , it is more convenient to compare the total Q-factor

$$\tilde{Q}_{\mu} \equiv \frac{Q_{\mu} Q_{\text{abs}}}{Q_{\mu} + Q_{\text{abs}}} \quad (9)$$

with Q_{abs} . Thus $\tilde{Q}_{\mu} \approx Q_{\text{abs}}$ for modes with $Q_{\mu} \gg Q_{\text{abs}}$ and $D_{\text{th}}^{\mu} \propto \tilde{Q}_{\mu}^{-1}$.

For a range of pump power $0 < D_0 < 4D_{\text{abs}}$, we know from equation (5) that the optimally outcoupled mode has $Q_{\mu} \approx Q_{\text{abs}}$, or equivalently, $\tilde{Q}_{\mu} = Q_{\text{abs}}/2$. Among the 56 modes within the full-width at half-maximum of the gain curve, there are two candidates close to this requirement: one with angular momentum $m = 25$ and outcoupling $Q_{\mu} \approx 1.05 \times 10^4$ and another with $m = 26$ and $Q_{\mu} \approx 2.18 \times 10^4$ (Fig. 3b). Because the former is closer to the gain centre $\omega_a R = 20.28$, it has a lower threshold $2.5D_{\text{abs}}$ (versus $2.88D_{\text{abs}}$) (Fig. 3a), and it is chosen as the target mode.

The spatial profile of mode $m = 25$ is shown in Fig. 3c. Like other whispering-gallery modes, it displays a maximum intensity on the ‘caustic’, a circle of radius $R_c \approx R \sin \chi_{\mu}$ (ref. 29), so a ring-shaped pump profile/contact geometry was chosen (Fig. 3c). The numerically calculated threshold of this mode (obtained by solving the full set of nonlinear equations (1)) is reduced by a factor of 5.52 when compared with the threshold of the same mode in the uniformly pumped case (Fig. 3a), agreeing with the $\text{Re}[f_{\mu}]$ -factor in the pump focusing effect discussed in the previous section, ‘Mode discrimination by spatially selective pumping’. For the higher-Q

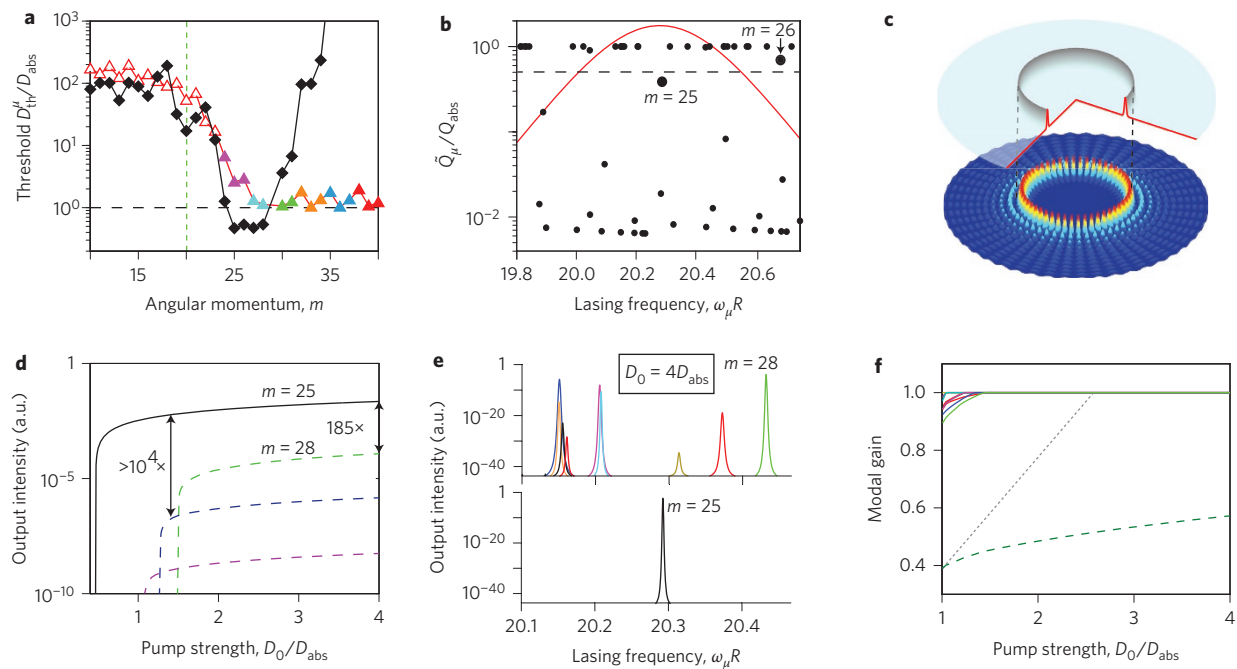


Figure 3 | Characteristics of a laser with $n_c = 3.3 + 10^{-4}i$, $\omega_a R = 20.28$ and $\gamma_1 R = 0.5$. **a**, Single-mode thresholds versus m for uniform (triangles) and selective (diamonds) pumping. Colour-grouped triangles indicate modes with the same radial quantum number η and the vertical line marks the mode with $\chi_{\mu} = \chi_c$. **b**, Total Q-factors versus mode frequency shown under the gain curve. The dashed line indicates Q_{opt} . **c**, Intensity distribution of mode $m = 25$ and the corresponding pump profile. **d**, Output intensity (in arbitrary units (a.u.)) versus pump for the selectively pumped laser (solid line) and the three strongest modes of the uniformly pumped laser (dashed lines). **e**, Spectrum at $D_0 = 4D_{\text{abs}}$ with uniform pumping (top) and selective pumping (bottom). Linewidths are schematic. **f**, Solid lines show modal gain for the uniformly pumped laser^{11,12}. The dotted line indicates that $m = 25$ turns on at $D_0 = 2.58D_{\text{abs}}$ in the absence of modal interactions (dotted line) and well outside the shown pump range if modal interactions are included (dashed line).

modes, the caustic shifts gradually outward with m , and results in a reduced overlap with the selected pump profile. For example, mode $m = 31$ at $\Omega_{\mu}R = 20.53$ exhibits an increased threshold, by a factor of 5.31, which is again well captured by $\text{Re}[f_{\mu}]^{-1} = 5.38$. The thresholds of modes with even higher m increase further and they can be safely discarded when lasing is concerned.

We first discuss the full nonlinear lasing solution with uniform pumping for $0 < D_0 < 4D_{\text{abs}}$, shown in Fig. 3d for the modes with the strongest output powers. The output spectrum measured in the far field at $D_0 = 4D_{\text{abs}}$ is shown in Fig. 3e (upper panel), with ten modes lasing. Mode $m = 28$ with $Q_{\mu} = 5.66 \times 10^5$ has the strongest output and the optimally outcoupled mode $m = 25$ is fully suppressed by nonlinear spatial hole burning interactions, regardless of how hard we pump. Such insight is very hard to gain with Maxwell–Bloch simulations. In the formalism discussed in the Supplementary Information we have direct access to the exact evolution of modal gain for modes, even below their lasing thresholds; this is shown in Fig. 3f, which reveals that the modal gain experienced by mode $m = 25$ is strongly nonlinear and saturates as more and more modes turn on.

For the proposed ring-shaped pump, the laser remains single-mode up to $D_0 = 4D_{\text{abs}}$. Remarkably, the total output power contained in this single mode is 185 times the total power delivered by the ten lasing modes with uniform pumping. Considering that the total input power is identical, just distributed differently in the plane of the cavity, this implies that mode competition via spatial hole burning interactions can reduce the power-efficiency of lasers substantially. We find this to be a very general statement that is valid for any laser cavity or gain mechanism, as discussed in Supplementary Section 2.

Discussion and conclusion

The enhancement of total output power with selective pumping proposed in this Article is universal. It derives from three

effects: (1) much improved outcoupling due to the particular mode selected, (2) threshold reduction due to ‘pump focusing’ and (3) reduced gain competition with fewer modes. There will be more total output power at a given input power regardless of the cavity shape and gain mechanism, if the pumping distribution is chosen appropriately. When complemented with active-region quantum design^{3,30–32}, spatially selective pumping can provide devices operating at optimal operation conditions and deliver maximal power under given fabrication and pump power constraints.

The precise value of the power enhancement will depend on the particular cavity geometry and pump power. The cases of a stadium-shaped resonator and a ridge-cavity microlaser are discussed in Supplementary Section 2, demonstrating the universality and robustness of our approach. Our general conclusion is that the largest enhancement factors can be expected at lower pump powers and for larger cavities.

As mentioned, our formalism and conclusions are also largely independent of the gain mechanism. We investigate this issue for a quantum cascade laser gain medium as well as for an optically pumped four-level gain medium in Supplementary Section 2. In certain solid-state lasers, the presence of significant current-spreading can result in a sub-optimal distribution of current in the active region, reducing expected output power enhancement factors. This can be alleviated by engineering the contacts through which the current is injected such that spreading is taken into account. We investigate the robustness of our results to current spreading in Supplementary Section 3.

Finally, we note that our conclusion implies that the deliberate introduction of loss in selected areas of a laser can ultimately increase the power-efficiency of the device. Thus, optical loss, which is traditionally avoided, can be a powerful tool when judiciously employed. Manipulation of the real part of the index of refraction of optical systems is today the cornerstone of modern

photonics and quantum optics. The type of gain/loss engineering demonstrated here and its electrical (or optical) control promises a new and extended design space, the full potential of which is to be explored in the coming years.

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Author contributions

L.G., O.M. and H.E.T. developed the theory and carried out the analytical calculations. L.G. and O.M. carried out the numerical calculations. H.E.T. supervised the project. All authors discussed the results and wrote the manuscript.

Additional information

Supplementary information is available in the [online version](#) of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to H.E.T.

Competing financial interests

The authors declare no competing financial interests.