

On-chip quantum simulation with superconducting circuits

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Using a well-controlled quantum system to simulate complex quantum matter is an idea that has been around for 30 years and put into practice in systems of ultracold atoms for more than a decade. Much recent excitement has focused on a new implementation of quantum simulators using superconducting circuits, where conventional microchip fabrication can be used to take design concepts to experimental reality, quickly and flexibly. Because the quantum ‘particles’ in these simulators are circuit excitations rather than physical particles subject to conservation laws, superconducting simulators provide a complement to ultracold atoms by naturally accessing non-equilibrium physics. Here, we review the recent wealth of theoretical explorations and experimental prospects of realizing these new devices.

In the context of quantum mechanics, it is common that physical problems of a seemingly simple nature turn out to be extremely difficult, if not impossible, to simulate on a classical computer¹. Examples are plenty, going far beyond the paradigmatic study of quantum phase transitions², and include a wide range of challenges such as computing molecular energy levels^{3,4}, predicting the critical temperatures in superconductors, and understanding matter in a neutron star. The same mechanism that drives emergent behaviour—the exponential proliferation of quantum states characterizing a large system—is also what makes the simulation of it on a classical computer intractable. Quantum simulation fights fire with fire, in a manner of speaking, employing a controlled quantum-mechanical device to mimic and investigate other quantum systems.

The idea of quantum simulation is not tied to any particular physical implementation⁵. Indeed, the earliest realizations of systems exhibiting these characteristics range from ultracold atoms in traps and optical lattices^{6–8}, measurement-based linear optics^{9,10}, and trapped ions^{11,12}, to Josephson-junction arrays^{13–15}. Proposals also exist for electrons in quantum-dot arrays¹⁶ and on the surface of liquid helium¹⁷.

The focus of this paper is to review the potential of superconducting circuits^{18–20}, for which experiments have demonstrated manipulation and measurement at the level of a few qubits and microwave photons, as quantum simulators. Superconducting-circuit-based quantum simulators stand apart from the systems considered as quantum simulators so far, both because of their flexibility in fabrication and their suitability for non-equilibrium simulation. These circuits are fabricated with optical and electron-beam lithography and can therefore access a wide range of geometries for large-scale quantum simulators. Moreover, because the ‘particles’ being simulated are just circuit excitations, particle number is not necessarily conserved. Unavoidable photon loss, coupled with the ease of feeding in additional photons through continuous external driving, makes such lattices open quantum systems, which can be studied in a non-equilibrium steady state. Thus, the physics that can be accessed with these open quantum simulators is different from what is typically studied, for example, in ultracold-atom experiments.

Building blocks of superconducting quantum simulators

Superconducting circuits provide a number of useful building blocks for quantum simulation, including harmonic oscillators and different qubit-forming anharmonic multi-level systems^{18–21}. Superconducting qubits come in a variety of types, often classified as charge^{22–26}, flux^{27–29}, or phase qubits³⁰, depending on the underlying circuit and methods of coupling; in their essence, however, they are all similar (Box 1). Today, such qubits can reliably achieve coherence times on the order of a few microseconds with high yield³¹, making fabrication of large systems a feasible goal. Qubits can be coupled to neighbouring qubits through mutual capacitance^{32,33} or inductance³⁴, through special elements allowing for tunable coupling^{35–41}, or over long distances by using a shared photonic mode^{42–44}.

Photonic modes can be realized with on-chip microwave resonators, essentially Fabry–Perot type cavities made from finite sections of transmission line. A qubit coupled to such a transmission-line resonator^{21,42} can faithfully realize the Jaynes–Cummings model (Fig. 1):

$$H^{\text{JC}} = \omega_r a^\dagger a + \varepsilon \sigma^+ \sigma^- + g(a \sigma^+ + a^\dagger \sigma^-)$$

Here, ω_r and ε are the photon and qubit excitation frequencies, and $a^\dagger$, a , σ^+ and σ^- denote the corresponding raising and lowering operators. As the lateral dimension of such transmission lines is typically a few micrometres and, thus, much smaller than one wavelength, the electric field in the cavity is relatively large and strong dipole coupling between qubits and photons can be readily achieved. Strong coupling cavity quantum electrodynamics (QED) has fuelled a great deal of research in superconducting quantum systems over the past decade²¹, and is a key component of many of the proposed quantum simulators.

One of the primary advantages of using superconducting circuitry is the great flexibility afforded by the nature of a nanofabricated system. Nearly every parameter involved is widely tunable with conventional lithography. Qubit frequencies can be readily set anywhere from 2 to 15 GHz, and couplings can range from a few kilohertz to nearly 1 GHz, merely by changing geometries. Capacitors at the ends of the cavity control photon leakage rates and, in multi-cavity systems, the hopping rates between neighbouring

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Box 1 | Working principles of superconducting qubits.

Every ‘good’ qubit requires a well-defined ground state that can serve as the logical-zero state, as well as an excited state that can serve as the logical-one state. For quantum systems with multiple excited states, an anharmonic spectrum is needed to enable operations that restrict probability amplitudes to only two select states. Different types and designs for superconducting qubits exist, but they all rely on three simple features afforded by superconductivity. First, as electrons pair up to form Cooper pairs, they can condense into a single, energetically well-defined ground state to serve as the logical-zero state. Second, currents in the superconductor are not subject to Ohmic dissipation. Third, superconducting tunnel junctions (Josephson junctions) provide a truly nonlinear circuit element—essentially, a nonlinear inductor—that renders the energy spectrum anharmonic, as required for selective addressing.

Different qubit designs create a variety of such energy spectra, but the nonlinearity is always introduced by the Josephson junction. The transmon^{25,26} is perhaps the superconducting qubit

simplest to understand. It is essentially an LC-oscillator with a weak nonlinear inductor that prevents the energy spectrum from being perfectly harmonic. Other designs rely on the discreteness of Cooper pairs tunnelling across the junction^{22,23}, a current-biased junction³⁰, or flux threading a superconducting loop^{27–29}.

All superconducting qubits can be controlled in similar ways. Few-nanosecond pulses of microwave radiation can drive Rabi oscillations between qubit levels $|0\rangle$ and $|1\rangle$, creating quantum superpositions along the way. Qubit parameters can often be tuned *in situ* as well. In many schemes, external magnetic fields are used as a method for tuning the qubit transition frequencies^{24,27–29}. In some devices, the strength of coupling between two qubits, or between a qubit and a microwave cavity, can be tuned in a similar way^{35–41}. As only very small currents in local wires are necessary to provide sufficient flux, these controls can be performed on a nanosecond timescale. For further details, see, for example, refs 18–20.

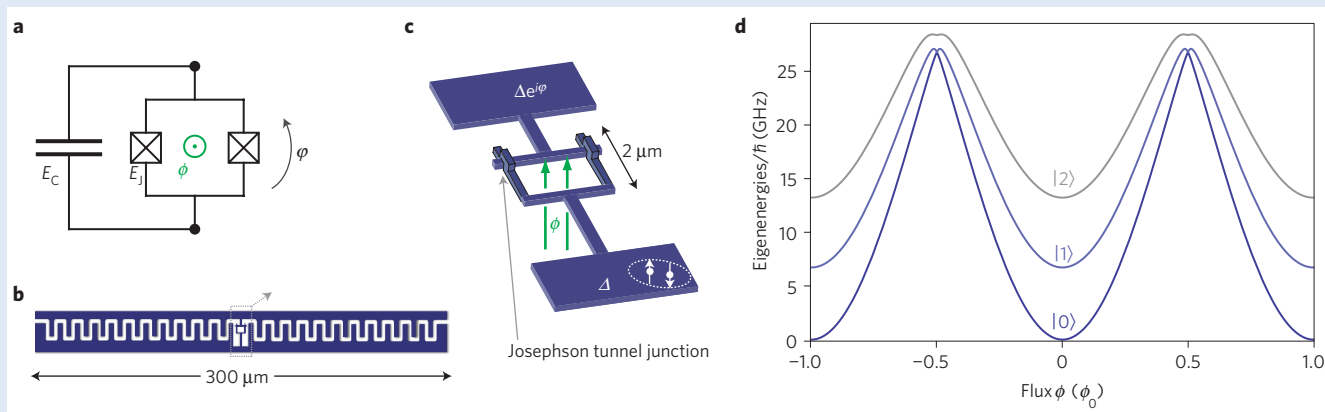


Figure B1 | Cooper-pair box. The Cooper-pair box is one of the simplest tunable superconducting qubits. **a**, Its circuit consists of two superconducting islands which are connected by two Josephson junctions forming a superconducting quantum interference device (SQUID). **b**, The circuit is patterned onto a chip, typically by means of double-angle electron-beam evaporation of aluminium to obtain Al-AIO_x tunnelling junctions, shown magnified in **c**. **d**, Energy levels of the lowest three Cooper-pair-box eigenstates as a function of external flux. (In this example, Josephson and charging energies are chosen as $E_J/\hbar = 30$ GHz and $E_C/\hbar = 0.2$ GHz, respectively; the offset charge has been fixed to $n_g = 1/2$.)

cavities. Even photon hopping with complex-valued amplitudes is possible when intermediate Josephson coupling elements are used to break time-reversal symmetry⁴⁵.

Fabrication of large-scale systems is not expected to be substantially harder than the fabrication of the small superconducting circuits already in use. Even lattices with intricate geometrical structure will be within reach of established fabrication techniques: essentially, any two-dimensional structure can be built, and the small size of elements allows for hundreds of elements on a single chip. It is easy to imagine regular one- and two-dimensional lattices, but even lattice structures with toroidal topology or fractal dimensions are conceivable by appropriately routing circuit connections between the boundaries of the lattice⁴⁶. Complete control over these large systems may still be a daunting task. Yet, many of the proposed large-scale simulators require only very simple interactions, and thus should be within experimental reach with present-day technology.

Precursors of superconducting quantum simulators

Nearly all of the work that has been done with superconducting qubits so far can be viewed as a form of quantum simulation at

some level. In essence, a superconducting qubit consists of a very large number of atoms, acting in concert to mimic the behaviour of a single spin system. Typically, this artificial-atom picture simply implies a series of anharmonic energy levels. However, in some recent work, superconducting qubits have been used in a deeper way to emulate the features of a true spin system. By rotating a simulated spin around a closed path on the Bloch sphere, a geometric Berry's phase is acquired, proportional to the angle subtended by the path⁴⁷. For a spin-1/2 system, a 4π rotation is needed to return to the original state³⁶.

Larger spins can be simulated using multiple levels of a superconducting qubit⁴⁸. In particular, a qubit with $2S+1$ levels can be used to simulate a spin S system. Because qubits are anharmonic, it is possible to selectively address each transition in the multilevel system; by driving these transitions in concert with pulses at $2S$ different microwave frequencies and powers, one can reproduce the Hamiltonian of, and thus emulate, a larger spin rotating in a magnetic field. By effectively rotating a larger spin through a closed path, Berry's phase is again acquired and is proportional to the spin quantum number S , as was observed for $S = 0, 1/2, 1$, and $3/2$ emulation. The even parity of integer spins and odd parity of

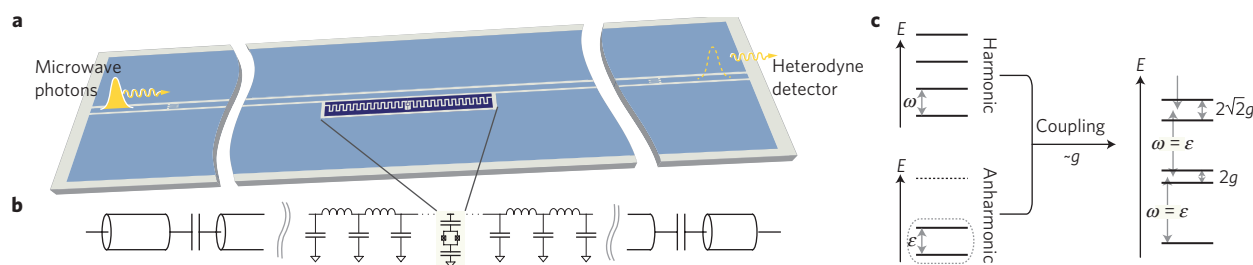


Figure 1 | Elementary circuit-QED building block for a quantum simulator. **a**, Standard fabrication techniques are used for patterning the simplest circuit-QED elements onto a chip: transmission-line resonators, superconducting qubits, and coupling capacitors. **b**, Capacitive coupling between a qubit and a resonator produces the simplest model for a single-lattice site. **c**, The Jaynes–Cummings model.

half-integer spins under 2π rotations can be directly observed for these simulated spins.

Simulations of small spin chains are also possible by directly coupling a number of superconducting qubits⁴⁹. In this experiment, a chain of eight ferromagnetically coupled spins in a one-dimensional chain was simulated with uniform coupling between nearest-neighbour spins. The ends of the chain were polarized in opposite directions, and an effective magnetic field gradient was induced by locally tuning each qubit transition energy, preferentially shifting the resulting domain wall towards one end of the chain. The system was annealed to the ground state, and reached the expected state on a timescale consistent with quantum annealing.

As well as simulating spin physics, a great hope of superconducting circuits is that they can be used to simulate condensed-matter physics with photons and polaritons far from equilibrium. In recent experiments in this direction, strong qubit-mediated photon–photon interactions have been observed using only a few quantum elements; these experiments include direct spectroscopy^{50,51}, collapse-and-revival experiments⁵² and correlation measurements⁵³. A single-qubit cavity system with large interactions has been used to emulate electronic transport in a quantum dot, with the typical control knobs of source–drain voltage and gate voltage mapped onto the spectral properties of a photonic bath. Wideband incoherent radiation replaces the Fermi sea in the source lead, with the bandwidth giving the effective source–drain voltage. As the bandwidth of incident radiation is increased, a staircase in transmitted power emerges, matching expectations from more conventional transport experiments involving interacting particles⁵⁴.

Two such circuit-QED systems can be coupled together to build a system that emulates a Josephson junction for photons with the Hamiltonian⁵⁵:

$$H = \sum_{i=L,R} H_i^{\text{JC}} - J(a_L^\dagger a_R + a_R^\dagger a_L)$$

where H_i^{JC} denotes the Jaynes–Cummings system on site $i = L, R$, and $a_i^\dagger$ and a_i are the photon creation and annihilation operators for site i . Here, one can begin to study the emergence of correlated behaviour, because the system is expected to undergo a non-equilibrium localization transition from a regime where the initial photon population imbalance between two resonators coherently oscillates between the two resonators (delocalized regime) to another regime where it becomes self-trapped (localized regime) as the photon–qubit interaction is increased beyond a critical value $g_c(J)$. This transition is driven by the competition between tunnelling and on-site interaction. Furthermore, because of photon leakage and qubit dissipation, this is an inherently dissipative system. The effective interactions in the Jaynes–Cummings Hamiltonian are weaker at higher photon numbers; therefore, dissipation favours the localized regime and can give rise to dynamical switching from the delocalized to the localized

regime. A numerical analysis of the master equation of the system shows that the localization transition is not washed out by quantum fluctuations, even down to an initial occupation as small as 20 photons, and should be readily accessible in experiments⁵⁵.

From here, the realization of circuit-QED arrays with a few resonators and qubits is relatively close. Their prominence in the recent body of literature^{56–64} is owed in part to their tractability in terms of brute-force numerics. Exact solutions for time evolution, steady states, and correlators of small-size systems have been obtained this way. In many cases, they already capture traces of the quantum phase transition or strongly correlated dynamics expected for the infinite-size system, even with as few as four lattice sites⁵⁸. These will provide an invaluable testbed for future experiments before moving to larger superconducting circuit networks.

From an experimental standpoint, moderate increases in the system size (that is, in the number of superconducting qubits and resonators) can be obtained naturally by extending samples currently in use in experiments aimed at quantum computation²¹. First experiments with such ‘mesoscale’ samples are currently underway. Beyond providing a proof of principle, they will allow characterization of the system parameters crucial for all future steps, and gather statistics on possible disorder in the system parameters caused by minor imperfections in fabrication.

The computationally accessible mesoscale regime is important from the point of view of benchmarking experimental systems. As systems grow, however, the exponential increase in Hilbert-space dimension may create theory ‘badlands’, where the computational cost for brute-force numerical solutions exceeds all reasonable limits, and yet system size remains too small for a statistical description based on the thermodynamic limit. This challenge of the mesoscale has been a central theme in condensed matter physics^{65,66}, nuclear physics and quantum chemistry of larger molecules, and gave birth to the statistical theory of mesoscopic systems^{67,68}. Incidentally, quantum simulation has recently been suggested as a practical way to tackle the challenge of predicting molecular spectra^{3,4}. Indeed, this computational intractability is the very reason why quantum simulators are necessary, although it renders performance verification extremely difficult.

Simulation with circuit-QED arrays

In large lattices of superconducting resonators, far beyond the mesoscale regime, it again becomes possible to develop an intuition for what a quantum simulator might reveal. Still, large interacting photon lattices challenge our understanding and modelling of strongly correlated systems, their quantum phases and their dynamics^{57,58,69–72}. These lattices ideally illustrate the potential of circuit-QED arrays for the purpose of quantum simulation, and continue to provide new impulses for the development of theoretical techniques capable of describing strongly correlated systems both in and out of equilibrium.

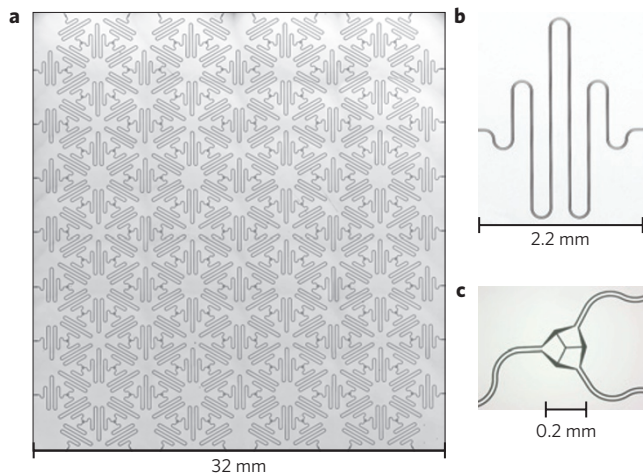


Figure 2 | Cavity lattice for quantum simulation. **a**, More than two hundred 7 GHz microwave cavities are coupled in a Kagome lattice, a natural two-dimensional lattice for these long, one-dimensional structures. To provide the necessary photon–photon interaction, qubits must be added to each cavity using an additional lithography layer. **b**, Each individual transmission-line cavity is nearly 10 mm long and only 20 μm across, and is folded to pack cavities densely on a chip. **c**, At each end, the cavities are capacitively coupled to two neighbours, enabling photon hopping. The symmetry of this three-way capacitor ensures uniform hopping rates throughout the array. (Device image: courtesy of D. Underwood and A. A. Houck.)

As qubits and cavities are made lithographically, it is possible to fabricate large arrays to observe many-body physics of interacting polaritons. With a 32 mm $\times$ 32 mm sample, it is feasible to couple over 200 cavities in a two-dimensional lattice. This number can easily be extended to more than 1,000 cavities on a full two-inch wafer. Disorder in the cavities alone can be small, on the order of a few parts in 10^4 , because each cavity is ~ 1 cm long and optical lithography is typically precise to the level of only a few micrometres. Preliminary experimental work suggests that this is indeed feasible (A. A. Houck, private communication). The geometry of transmission-line cavities also dictates the types of lattice that are natural. Each ‘site’ in a simple lattice is essentially two-dimensional, with two distinct endpoints where photons may enter. The real-space sketch of a resonator lattice,

depicting resonators as a line segment, is thus the dual, or line graph of the actual lattice. The 200-cavity sample in Fig. 2, for instance, is an array of resonators forming a honeycomb pattern. The resulting lattice is the kagome lattice (of which the honeycomb lattice is the line graph). Qubits, not yet included in the sample of Fig. 2, can be added in a further step of electron-beam lithography. Without individual tunability of qubit frequencies, disorder in qubit parameters must be expected to be larger than cavity disorder. However, because a strong photon–photon interaction can be realized when qubits are far off resonance, the effects of this disorder will be mitigated, and many-body behaviour could potentially be observed using only global control⁵⁴.

From a theoretical standpoint, the infinite-system limit is particularly appealing owing to the availability of tools that are appropriate for large systems, ranging from ‘pedestrian’ mean-field theory to powerful methods such as variational cluster techniques. Depending on the specific method, approximate—and sometimes exact—results can be obtained that reveal important properties of the ground state, the elementary excitations, or relevant correlation functions. The paradigmatic model exhibiting a quantum phase transition akin to the superfluid–Mott insulator transition in the Bose–Hubbard model^{2,73} is the Jaynes–Cummings lattice with nearest-neighbour photon hopping (at rate κ):

$$H = \sum_j H_j^{\text{JC}} - \kappa \sum_{\langle i,j \rangle} (a_i^\dagger a_j + a_j^\dagger a_i) \quad (1)$$

Despite the apparent lack of interaction terms such as $(a_i^\dagger a_j)^2$, the Jaynes–Cummings lattice is an interacting model equivalent to a Bose–Hubbard-like model with two species of bosons, one of which has an infinite Hubbard parameter $U \rightarrow \infty$ to reproduce the pseudo-spin 1/2. Analogous to the Bose–Hubbard model, the key physics of the Jaynes–Cummings lattice consists of the competition between polariton delocalization, induced by photon hopping, and on-site interaction, which tends to freeze out hopping and localize polaritons.

For an array of equivalent cavities, the Jaynes–Cummings lattice described by equation (2) has a global $U(1)$ symmetry, so that the total polariton number $N = \sum_j (a_j^\dagger a_j + \sigma_j^+ \sigma_j^-)$ is conserved. It is convenient to employ a grand-canonical description, where the Hamiltonian is replaced by $\mathcal{H} = H - \mu N$, where μ denotes the chemical potential, although experimental procedures for realizing such an effective chemical potential will need to be developed. Much of the qualitative physics contained in the Jaynes–Cummings

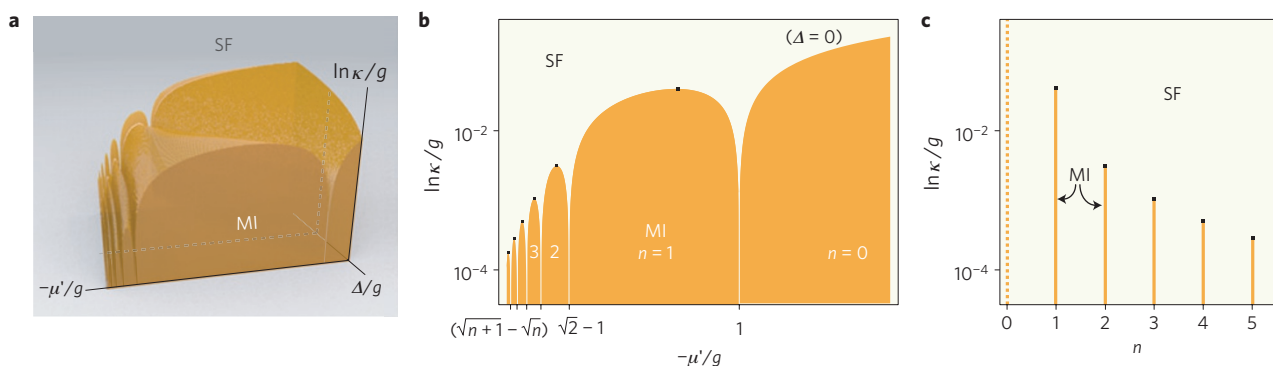


Figure 3 | Superfluid-to-Mott-insulator transition in the Jaynes–Cummings lattice. Polaritons in an infinite lattice of Jaynes–Cummings sites with nearest-neighbour photon hopping can undergo a quantum phase transition from a compressible superfluid (SF) phase to Mott insulating (MI) phases with integer polariton number n on each site. **a**, The critical region can be accessed by tuning the photon hopping κ , the qubit–resonator detuning Δ , or the chemical potential μ (here denoted as $\mu' = \mu - \omega_r$). **b**, The transition follows the universality class of the Bose–Hubbard model, including the characteristic switch of the dynamic critical exponent at the multicritical points located at the tips of each lobe. **c**, Within the canonical ensemble, the total polariton number remains fixed, Mott lobes reduce to line segments at integer filling factor in the phase diagram.

Box 2 | Basics of quantum dynamics in open systems.

Superconducting circuits are never completely isolated. In part, this coupling between the ‘system’ of interest (for example, a circuit-QED lattice to be used as a quantum simulator) and the ‘environment’ is desired: it allows for the experimental control and measurement of the system state. Small contributions from coupling between the system and environmental degrees of freedom beyond the control of the experimentalist are unavoidable, and are sources of dephasing and energy relaxation, including spurious photon leakage. Superconducting circuits thus offer a versatile platform to realize open quantum systems¹⁰². A natural way to probe such a system proceeds by continuously driving the system with a microwave generator. After an initial time period with transient behaviour, the system settles into a non-equilibrium steady state with a mean polariton number that can be controlled by varying the drive strength.

An open quantum system can be described in terms of ρ , its reduced density matrix. ρ is obtained from the total density matrix ρ_{tot} by averaging over the environment, which can be expressed as a partial trace: $\rho = \text{tr}_{\text{env}} \rho_{\text{tot}}$. Under appropriate conditions of a separation of timescales between the reservoir and the system dynamics, the time evolution of $\rho = \rho(t)$ is captured by a Lindblad master equation that relates $\dot{\rho}$ to two contributions:

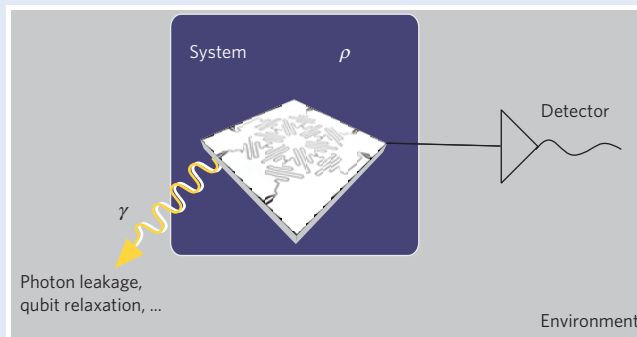
$$\dot{\rho}(t) = -\frac{i}{\hbar} [H_{\text{sys}}, \rho] + \sum_l \gamma_l \mathcal{D}[C_l] \rho(t) \quad (2)$$

The first term describes the ordinary unitary evolution under the system Hamiltonian H_{sys} . The second term expresses the influence of the environment in terms of decoherence rates γ_l and dissipators $\mathcal{D}[C_l] \rho \equiv ([C_l \rho, C_l^\dagger] + [C_l, \rho C_l^\dagger])/2$. The operators C_l determine the nature of the decoherence process. Common examples include the photon-annihilation and qubit operators $C_l = a$, σ^- , and σ^z , which capture photon loss, qubit relaxation and qubit dephasing, respectively.

The steady-state solution of equation (1), defined by $\dot{\rho} = 0$ can be reduced to a linear algebra problem as follows. The Hilbert space for a system of N_q qubits and N_r resonators has an orthonormal Fock-state basis with states $|n_1, n_2, \dots, n_{N_r}; \sigma_1, \sigma_2, \dots, \sigma_{N_q}\rangle$,

where n_j is the number of photons in resonator j and $\sigma_i = \uparrow, \downarrow$ is the state of qubit i . To solve for the steady state, this basis must be truncated. The simplest (but not smartest) truncation scheme consists of limiting the maximum photon number in each resonator to n_{cutoff} . The dimension of the resulting truncated space is $M = 2^{N_q} (n_{\text{cutoff}} + 1)^{N_r}$. Re-arranging the matrix elements $\rho_{mn} = \langle m | \rho | n \rangle$ to form a so-called coherence vector $\vec{\rho} = (\rho_{11}, \rho_{12}, \dots)$ with M^2 components, the steady-state equation can be written as $L\vec{\rho} = 0$. In principle, the steady-state solution can thus be obtained as the eigenvector of the $M^2 \times M^2$ matrix L —which is not necessarily Hermitian—corresponding to eigenvalue $\lambda = 0$. The steady-state expectation value of an observable of interest, O , can then be calculated in the usual way using $\langle O \rangle = \text{tr}[\rho O]$.

It is essential to emphasize the enormous matrix sizes obtained even for rather small system size. For a system composed of $N_r = 4$ resonators, $N_q = 4$ qubits, and using a modest photon-number cutoff of $n \leq n_{\text{cutoff}} = 3$, the matrix L has dimensions of roughly 16 million $\times$ 16 million. A slight decrease in matrix size can be achieved by clever cutoff schemes and exploiting that the Hermiticity and normalization of ρ lead to a reduction in the number of independent matrix elements ρ_{mn} . However, straightforward brute-force numerics quickly becomes unmanageable, and theory will have to identify appropriate approximation techniques suitable for strongly correlated open quantum systems.



lattice can be understood from a simple mean-field decoupling of the photon hopping. The resulting effective Hamiltonian for a single site, given by

$$H_{\text{mf}} = -\mu' a^\dagger a + (\Delta - \mu') \sigma^+ \sigma^- + g(a^\dagger \sigma^- + a \sigma^+) - \kappa z_c (a^\dagger \psi + a \psi^* - |\psi|^2)$$

allows for spontaneously broken $U(1)$ symmetry, which is reflected in the phase of the complex-valued order parameter $\psi = |\psi| e^{i\varphi} = \langle a \rangle$. The coordination number of the lattice (which is the number of nearest neighbours to any given site) is denoted by the integer z_c , $\mu' = \mu - \omega_r$ is the shifted chemical potential, and $\Delta = \varepsilon - \omega_r$ the detuning between qubits and resonators.

The resulting mean-field phase diagram^{60,69,74–76} is shown in Fig. 3. In striking resemblance to the Bose–Hubbard model², the Jaynes–Cummings lattice undergoes a quantum phase transition from a superfluid of delocalized polaritons with $|\psi| > 0$ to Mott-insulating phases. In each Mott phase, the average number of polaritons per site, measured by $n = a^\dagger a + \sigma^+ \sigma^-$, is fixed to an integer—Mott phases are incompressible—and the order parameter ψ vanishes.

The similarity between Bose–Hubbard and Jaynes–Cummings lattice is no coincidence. The key competition between an

on-site repulsive interaction and hopping between sites is quite independent of the detailed nature of the ‘particles’ (elementary bosons or polaritons, respectively). Large on-site repulsion generically inhibits hopping and, at integer filling ν , stabilizes states with a fixed number of particles per site, $|\Psi\rangle = \prod_{\text{site } j} (a_j^\dagger)^\nu |0\rangle$. By contrast, strong hopping favours delocalization and condensation of the particles into the zero-momentum state, $|\Psi'\rangle = (a_{\mathbf{k}=0}^\dagger)^N |0\rangle$. Effective-field theory^{73,75} and recent quantum Monte Carlo simulations⁷⁷ corroborate that the Bose–Hubbard model and the Jaynes–Cummings lattice fall into the same universality classes, resulting in identical behaviours at criticality.

For quantitative predictions, more sophisticated methods which account for correlations in the many-polariton system are necessary, and have led to a fairly detailed understanding of the Jaynes–Cummings lattice. Modest changes in the true location and shape of the phase boundary in two dimensions have been identified using strong-coupling theory⁷⁶, based on the linked-cluster expansion⁷⁸, and numerically with quantum Monte Carlo simulations^{77,79,80}. The nature of the transition differs from one-dimensional lattices, where it is of Berezinskii–Kosterlitz–Thouless type. This results in a more significant deformation of Mott-insulating lobes and the development

of cusps, observed in density-matrix-renormalization-group (DMRG) calculations^{81,82} and in works using the virtual-crystal-approximation (VCA) approach^{80,83}.

Fingerprints of the more subtle differences between the Bose–Hubbard model and the Jaynes–Cummings lattice are rooted in the composite nature of the polaritons. In the Mott phase, the joint contributions of qubits and photons are reflected in a doubling of excitation modes with easily distinguishable dispersions^{76,80}. In the superfluid phase, a ‘smoking gun’ observation might be an anomalous change in the dependence of the sound velocity on detuning, $c_s = c_s(\Delta)$, which is predicted to be monotonic for a polariton density of $n = 1$, but non-monotonic at larger densities⁸⁴. The flexibility to tune the system from resonant polaritons ($\Delta = 0$) to nearly decoupled subsystems composed of qubits and photons ($|\Delta| \gg g$), has the further benefit of enabling quantum simulation of spin systems such as isotropic and anisotropic Heisenberg models in one or two dimensions^{58,59,85–87}.

Harnessing the huge versatility of re-arranging lattice building blocks in the fabrication step, numerous modifications of the Jaynes–Cummings lattice are conceivable: changing the number of qubits inside each resonator, using multi-mode cavities, and lattices with alternating elements or different numbers of nearest neighbours. Such variations may lead to further interesting phenomena such as fermionization of photons^{62,88} and Luttinger-liquid physics⁸⁹. Even fractional-quantum-Hall physics in a bosonic setting—which has been proposed for ultracold atoms^{90,91} but has remained elusive so far—may be realizable with polaritons⁹². Concrete ideas for mechanisms to break time-reversal symmetry in circuit-QED lattices have recently been proposed and are being explored experimentally^{45,93}.

Possibly, the most intriguing physics of circuit-QED arrays is still to come. Accepting photon loss as inevitable, a natural setting for experiments is likely to be a steady state of the polariton lattice under continuous driving. The non-equilibrium aspect and necessity to develop and solve models for open quantum systems make this problem challenging for theory (Box 2). Numerically solving the steady-state equation $\rho = L\rho = 0$ is generally very difficult because of the enormous size of the superoperator matrix L . First steps in this direction were taken for small arrays of quantum-nonlinear cavities^{54,55,61–64}. The dynamics of larger arrays can be accessed through the representation of the density matrix as a matrix product operator⁹⁴. A viable alternative is using a mean-field approximation based on site-decoupling, which has been successfully employed to study equilibrium Hubbard models. In this spirit, ref. 95 employed a cluster mean-field approximation to discriminate Mott and superfluid phases of a polariton lattice through a quantum quench of the dissipative Bose–Hubbard model.

Future directions

With a growing number of theoretical ideas and analyses, the future looks bright for non-equilibrium quantum simulation with superconducting circuits. New ideas continue to emerge, even for relatively simple systems. One particular area beginning to bear fruit is that of superconducting qubits coupled to open transmission lines, where transport physics akin to those in quantum-impurity systems has been investigated experimentally⁹⁶, and there exist proposals for studying phenomena such as photonic bound states^{97,98} and the Kondo effect⁹⁹.

Although fabrication of large circuit-QED lattices does not pose fundamental problems, several challenges remain before such simulators can really become practical. An important aspect of experimental realizations consists of control and tunability of system parameters. In a large lattice, it is possible to tune all qubit frequencies simultaneously with an applied magnetic field. This affects the overall detuning, but does not remove disorder. To what

degree disorder will affect the expected phase transition in detail remains an open theoretical question. In smaller systems, local tuning of qubit frequencies and qubit–photon coupling strengths can be achieved by local flux-bias lines, and used to reduce disorder. As technology progresses, the size of systems where individual control can be exerted is likely to grow. Other parameters, such as the photon-hopping rate, are defined lithographically, and can be changed systematically from sample to sample.

Challenges also include the question of suitable techniques for readout and state preparation in large arrays. Potential answers are on the horizon: for preparation, there is hope that steady-state driving, rather than individual-site initialization, could be sufficient. In smaller systems, individual-site initialization should be possible using the techniques developed for single cavities⁵². On the measurement side, transport through a cavity array represents the easiest method of detection. Local measurements at the level of single lattice sites⁶³ enable, for instance, the gathering of photon-number statistics inside a single cavity¹⁰⁰. Moreover, an entire array of qubits could be coupled to a single bosonic mode, such as in a 3D cavity¹⁰¹, giving easy access to collective states. These future experiments with superconducting circuits will be full-fledged simulations of open quantum systems. They will give invaluable insight into the development of the systematic theory, which is currently lacking, and help improve our understanding of the interplay between non-equilibrium and strong correlations in many-body systems.

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Additional information

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