

Self-protected polariton states in photonic quantum metamaterials

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(Received 10 September 2013; published 10 February 2014)

We investigate the single-photon transport properties of a one-dimensional coupled cavity array (CCA) containing a single qubit in its central site by coupling the CCA to two transmission lines supporting propagating bosonic modes with linear dispersion. We find that even in the nominally weak light-matter coupling regime, the transmission through a long array exhibits two ultra-narrow resonances corresponding to long-lived, self-protected polaritonic states localized around the site containing the qubit. The lifetime of these states is found to increase exponentially with the number of array sites, in sharp distinction to the polaritonic Bloch modes of the cavity array.

DOI: [10.1103/PhysRevA.89.025801](https://doi.org/10.1103/PhysRevA.89.025801)

PACS number(s): 42.50.Ct, 32.80.Qk, 42.50.Nn, 42.50.Pq

Cavity QED studies the nonequilibrium dynamics of quantum emitters (atoms or qubits) coupled to discrete photon modes of an electromagnetic resonator. Such systems are of importance in the study of fundamental properties of open quantum systems, as well as for quantum information processing applications. Of particular interest is the strong-coupling regime of cavity QED achieved when a single excitation can be coherently exchanged between the qubit and a single photon mode before leaving the cavity. This regime is characterized by well-defined quasiparticles, loosely referred to as polaritons, that are adjustable mixtures of photonic and material excitations.

In recent years, we are witnessing the shift of focus towards larger cavity QED architectures which display collective effects due to the interaction of many photonic modes and qubits. Such collective effects can arise in a multitude of ways. Complex states of matter, such as atomic [1–3] and polaritonic [4,5] condensates, represent examples where many emitters are coupled coherently to a single cavity. Waveguide QED [6–19] and photonic impurity systems [20,21] constitute examples where single or few emitters interact with many optical modes. Recent theoretical work on cavity QED lattices [22–25] addressed systems that display interactions of many emitters with many modes.

In this paper, we investigate the strong-coupling physics and polariton formation when a single qubit is coupled to an open mesoscopic photonic system. From the perspective of the qubit, the mesoscopic system represents a dissipative electromagnetic environment featuring a spectral density that is highly structured. In the absence of additional qubit decay channels, long-lived polaritonic states typically are formed in such a system when the rate of coherent exchange between the qubit and the photonic modes is much faster than the photonic decay rate κ . Here, we show that light-matter interaction in a mesoscopic environment can result in polaritonic modes with a hugely enhanced lifetime. For this we consider a one-dimensional array of N coupled cavities containing a single qubit in its central site (see Fig. 1). This photonic quantum metamaterial (MM) [26,27] is coupled to waveguides that constitute the sole channel for dissipation and allow us to probe the system through photon scattering. Such a setup can be realized in circuit QED platforms with existing

technology [24,25,28,29]. Even in the regime of nominally weak light-matter coupling, we find, among other broader features, two transmission peaks of nearly vanishing linewidth, corresponding to *quasibound* photon-qubit states localized around the central cavity. These are finite-lifetime modes that derive from photon-qubit bound states when the cavity array extends to infinity [15–17]. As long as the hopping rate J between cavities is smaller than the qubit-cavity coupling g , the lifetime τ of these modes is shown to scale exponentially with the number N of array sites, $\tau \sim (g/J)^{N-1}/\kappa$, in clear distinction with the other polaritonic Bloch modes of the array. This result demonstrates the existence of interaction-induced, self-protected polariton states of different origin than the remaining photon-dominated Bloch modes of the lattice.

The combined system of quantum MM and waveguide is described by the Hamiltonian $H = H_{\text{MM}} + H_w + H_{\text{MM}w}$, where the first term describes the metamaterial in terms of a finite set of local bosonic- and qubit operators, a_j , $j = 1, \dots, N$ and $\sigma_i^\pm$, $i = 1, \dots, K$. Here, we consider as a metamaterial an array of N coupled cavities (N odd) with frequency ω_c containing one ($K = 1$) resonant two-level system with $\omega_q = \omega_c$ at its middle site $j = s$ [see Fig. 1(b); we set $\hbar = 1$],

$$H_{\text{MM}} = H_{\text{CCA}} + \omega_q \sigma^+ \sigma^- + g (\sigma^+ a_s + a_s^\dagger \sigma^-) \quad (1)$$

with

$$H_{\text{CCA}} = \omega_c \sum_{j=1}^N a_j^\dagger a_j - J \sum_{j=1}^{N-1} (a_j^\dagger a_{j+1} + \text{H.c.}) \quad (2)$$

The second term in H_{CCA} describes photon hopping between nearest neighbors at a rate J , with a bare photon bandwidth $4J \cos[\pi/(N+1)] \approx 4J - \mathcal{O}(J/N^2)$. The qubit couples to the cavity photon at site s with strength g . The other terms in H describe the kinetic energy of the waveguides,

$$H_w = -iv_g \sum_{\alpha=l,r} \int_{-\infty}^{+\infty} dx_\alpha \Psi_\alpha^\dagger(x_\alpha) \partial_{x_\alpha} \Psi_\alpha(x_\alpha), \quad (3)$$

and the metamaterial-waveguide coupling

$$H_{\text{MM}w} = \sum_{\alpha=l,r} \int_{-\infty}^{+\infty} dx_\alpha \delta(x_\alpha) V_\alpha \Psi_\alpha(x_\alpha) a_s^\dagger + \text{H.c.} \quad (4)$$

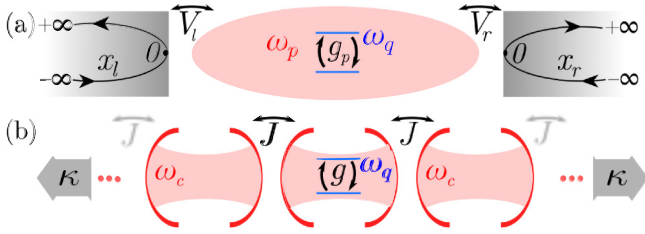


FIG. 1. (Color online) Scheme of a general quantum metamaterial (MM) with photonic modes ω_p (red) coupled to a qubit of frequency ω_q (blue) with strengths g_p (a). The MM is coupled to two waveguides with branch coordinates $x_{l,r}$. The case $x_{l,r} > 0$ describes left (right) propagating modes with group velocity v_g in the left (right) waveguide, while for $x_{l,r} < 0$ the propagation direction is reversed. Both waveguides couple to the MM at $x_{l,r} = 0$ via the photon hopping strength $V_{l,r}$. Panel (b) shows the specific MM considered in this paper, an array of N coupled cavities with frequency ω_c , photon hopping strength J , and a qubit in the central cavity with coupling strength g . The array couples to the waveguides via the outer cavities, leading to a characteristic photon escape rate $\kappa = |V_{l,r}|^2/v_g$.

Here, the operator $\Psi_\alpha(x_\alpha)$ destroys a photon with group velocity v_g in the left (l) or right (r) waveguide at the branch coordinate x_α [see Fig. 1(a)]. $H_{\text{MM}w}$ describes photon hopping with coupling constants V_α between the coupled cavity array (CCA) boundary sites and the waveguides (we defined $a_l \equiv a_1$ and $a_r \equiv a_N$). The linear dispersion model (3) is well suited for waveguides supporting transverse electromagnetic (TEM) modes, e.g., in circuit QED [30]. A fundamental mode with linear dispersion is found also in other bosonic wave-guiding systems, e.g., surface plasmons in metallic nanowires [31] and photonic crystal waveguides [32].

In order to calculate the transmission, it is convenient to write the Hamiltonian of the MM in its eigenbasis, i.e., $H_{\text{MM}} = \sum_n \Omega_n P_n^\dagger P_n$, with eigenvalues Ω_n and the projectors $P_n = |\text{vac}\rangle \langle n|$, which destroy an excitation in the eigenstates $|n\rangle$. Since the total Hamiltonian H commutes with the excitation operator $N_{\text{ex}} = \sum_{\alpha=l,r} \int_{-\infty}^{+\infty} dx_\alpha \Psi_\alpha^\dagger(x_\alpha) \Psi_\alpha(x_\alpha) + \sum_n P_n^\dagger P_n$, we can compose the one-excitation ansatz for the combined metamaterial-waveguide system in the form

$$|\phi_k\rangle = \left[\sum_{\alpha=l,r} \int_{-\infty}^{+\infty} dx_\alpha \phi_k^\alpha(x_\alpha) \Psi_\alpha^\dagger(x_\alpha) + \sum_n p_k^n P_n^\dagger \right] |\text{vac}\rangle.$$

Solving the Schrödinger equation, we obtain the Lippmann-Schwinger (LS) states for a MM of length $2L$,

$$\begin{aligned} \phi_k^l(x_l) &= e^{ik(x_l-L)} [\theta(-x_l) + r_k \theta(x_l)], \\ \phi_k^r(x_r) &= e^{ik(x_r-L)} t_k \theta(x_r), \end{aligned} \quad (5)$$

with energies $\epsilon_k = v_g k$ and the transmission and reflection amplitudes

$$t_k = \frac{-2i\beta}{\Gamma_l \Gamma_r + |\beta|^2} \quad \text{and} \quad r_k = \frac{\Gamma_l^* \Gamma_r - |\beta|^2}{\Gamma_l \Gamma_r + |\beta|^2}, \quad (6)$$

where

$$\Gamma_{l,r} = 1 + \frac{i}{2v_g} \sum_n \frac{|V_n^{l,r}|^2}{\epsilon_k - \Omega_n}, \quad \beta = \frac{1}{2v_g} \sum_n \frac{V_n^l [V_n^r]^*}{\epsilon_k - \Omega_n}. \quad (7)$$

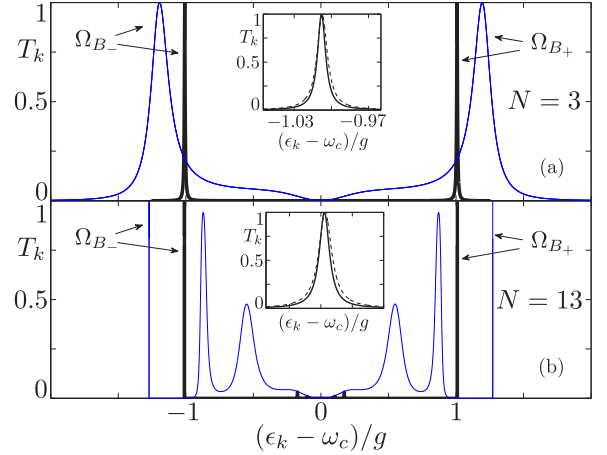


FIG. 2. (Color online) Transmission $T_k = |t_k|^2$ as a function of the photon energy ϵ_k for an array with $N = 3$ (a) and $N = 13$ (b) cavities containing a qubit in the central site, with hopping rate $J/g = 0.1$ (black, thick) and $J/g = 0.5$ (blue, thin) for $\kappa = g$. The ultranarrow resonances denoted by $\Omega_{B\pm}$ are associated with polaritonic quasibound states localized around the center site of the cavity array (see text). The insets show a zoom into the left ultranarrow resonance for $J/g = 0.1$. Dashed lines correspond to the analytic approximation in Eqs. (13) and (14). The other peaks ($N = 13$, blue and thin) are associated with overlapping photonlike resonances (hardly visible at $J/g = 0.1$).

The couplings $V_n^{l,r} = V_{l,r} u_n^{1,N}$ involve the photonic amplitudes $u_n^j = \langle \text{vac} | a_j | n \rangle$ at the edges. The probability amplitude for the excitation of the mode n in the metamaterial associated with the LS state k is given by

$$p_k^n = \frac{e^{-ikL}}{2(\epsilon_k - \Omega_n)} [V_n^l (1 + r_k) + V_n^r t_k]. \quad (8)$$

The results Eqs. (6)–(8) provide the general solution of the one-photon scattering problem for any kind of MM which is connected to two transmission lines; the specific properties of the MM at hand are encoded in the energies Ω_n and amplitudes $u_n^{1,N}$. For the MM in Eq. (1), the eigenvalues Ω_n and eigenstates $|n\rangle$ are calculated numerically. In Fig. 2 we show the transmission $T_k = |t_k|^2$ for a CCA with $N = 3, 13$ cavities; given the coupling g , we choose $V_{l,r} = V = \sqrt{v_g g}$, generating a “weak” coupling situation $\kappa = V^2/v_g = g$. In both cases we find two well-resolved peaks with ultranarrow linewidths. As we will explain below, these peaks describe high- Q quasibound polaritonic states localized around the central cavity. The linewidth is smaller for weaker hopping strength J (black, thick curves) and larger array size N [Fig. 2(b)]. Bound states [33] and the effect of quasibound states on transmission [34] were also found in mesoscopic electronic systems, specifically in the context of ballistic transport through narrow wires with stubs or side-coupled dots. Let us then analyze the properties of the quasibound polaritonic states for the present situation of a photonic metamaterial. To this end, we first find the eigenvalues and eigenfunctions of the metamaterial Eq. (1). Solving $H_{\text{MM}} |n\rangle = \Omega_n |n\rangle$ for a single excitation, one obtains a nonlinear equation for the

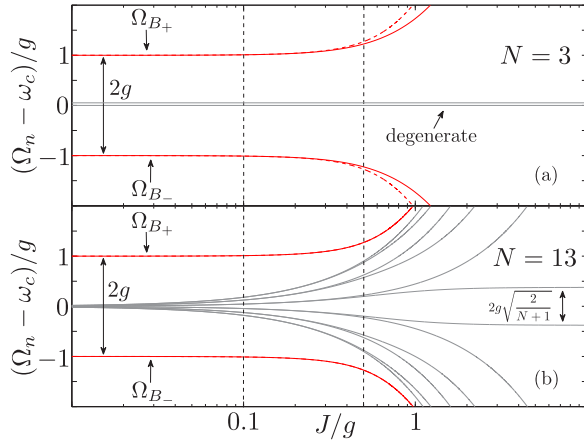


FIG. 3. (Color online) Eigenenergies of the quantum metamaterial in Eq. (1) as obtained from a numerical solution of Eq. (9) with $N = 3$ (a) and $N = 13$ (b) cavities plotted as a function of the effective hopping strength J/g . The red lines depict the energies of the out-of-band states, giving rise to the ultranarrow resonances seen in Fig. 2. (The vertical dashed lines mark the values of J/g considered there.) The red dashed lines correspond to the analytic approximation in Eq. (10), valid for a large number of cavities $N \gg 1$.

eigenvalues Ω_n ,

$$\frac{g^2}{\Omega_n - \omega_q} \sum_{p=1}^N \frac{|\alpha_p^s|^2}{\Omega_n - \omega_p} = 1, \quad (9)$$

with $\omega_p = \omega_c - 2J \cos[\pi p/(N+1)]$ the energies of the bare CCA and the photonic amplitudes $\alpha_p^s = \sqrt{2/(N+1)} \sin[\pi p s/(N+1)]$ evaluated at the site s of the qubit. When $|\Omega_n - \omega_c| < 2J$, there are $N-1$ in-band solutions, while for $|\Omega_n - \omega_c| > 2J$, the sum in Eq. (9) can be evaluated analytically for $N \gg 1$ and one obtains two out-of-band states with energies

$$\Omega_{B\pm} = \omega_c \pm \sqrt{2J^2 + \Omega^2} \quad \text{with} \quad \Omega^2 = \sqrt{4J^4 + g^4}. \quad (10)$$

The exact energies as obtained numerically from Eq. (9) are shown in Fig. 3 together with the analytic result in Eq. (10) (dashed lines). We observe that the bound-state energies $\Omega_{B\pm}$ agree with the position of the ultranarrow resonances in Fig. 2 up to a small dissipation-induced shift to be discussed later. The corresponding eigenfunctions describe entangled photon-qubit states, which are localized around the central cavity. In the weak-hopping regime ($J \ll g$) and for $N \gg 1$, we can find an approximate form of the wave functions by assuming periodic boundary conditions, yielding

$$|B_{\pm}\rangle = \frac{1}{\mathcal{N}} \sum_{j=1}^N e^{-|j-s|/\xi} (\alpha a_j^{\dagger} \pm g \delta_{js} \sigma_j^+) |\text{vac}\rangle \quad (11)$$

with the localization length

$$\xi = -1/\ln \eta, \quad \eta = \pm \sqrt{(\Omega^2 - g^2)/2J^2}. \quad (12)$$

Furthermore, $\alpha = \pm \sqrt{2J^2 + \Omega^2}$ and $\mathcal{N} = \pm \sqrt{f\alpha^2 + g^2}$, with $f = (1 + \eta^2 - 2\eta^{N+1})/(1 - \eta^2)$; these results are consistent with those obtained in Refs. [16,17], where one qubit-cavity system inside an infinitely extended CCA with periodic

boundary conditions was considered. In order to obtain experimentally measurable signatures of these self-protected polaritonic bound states, e.g., in circuit QED, the finite size and the coupling of the metamaterial to an external transmission line must be taken into account. As we will show below, the role of the waveguide continuum is important in defining the interesting properties of the bound states (11) in a realistic open system. Note that in circuit QED, excitation and readout can be achieved by making use of a *single-photon source* on one and a microwave detector on the other end of the two transmission lines enclosing the CCA [35].

We now analyze the transmission close to the bound states, i.e., assuming $\epsilon_k = \Omega_{B\pm} + \delta$, with $|\delta| \ll \Omega_{B\pm}$. In the weak-hopping regime ($J \ll g$), the transmission can be cast into a Lorentzian form,

$$T_k \approx \frac{W_{B\pm}^2}{[\delta \pm (\kappa/2g)W_{B\pm}]^2 + W_{B\pm}^2}, \quad (13)$$

describing the transformation of the bound states (11) into quasibound states with a lifetime $\tau_{B\pm} = 1/W_{B\pm}$ and a frequency shift $\mp(\kappa/2g)W_{B\pm}$ due to dissipation (we have dropped terms $\propto \kappa^2/g^2$). The Lorentzian line shapes compare well with the exact numerical result, as can be seen in the insets of Fig. 2. A central result of this work then is the narrow linewidth

$$W_{B\pm} \approx \frac{\kappa}{2} (J/g)^{N-1} + \kappa \mathcal{O}[(J/g)^{N+1}], \quad (14)$$

decreasing exponentially in N within the weak-hopping regime $J \ll g$; the narrow peaks describe two long-lived, self-protected polariton resonances that are formed by a qubit-induced localization of a photon. It turns out that within a circuit QED setup, an array of $N = 3$ cavities is already sufficient to support a long-lived polariton mode around its middle site. Such a three-resonator device is an architecture that is readily realizable today [24,25,28,29]; in these experiments, typical values for the lead coupling are $\kappa = 10 \text{ kHz} - 80 \text{ MHz}$, while the light-matter coupling g and photon tunneling J go up to a few hundred megahertz. The cavity- and qubit-decay rates are in the kilohertz regime and are thus small in comparison with g , J , and κ [25]; in our analysis, we have neglected such additional dissipative processes. Choosing $\kappa = 80 \text{ MHz}$, $g = 200 \text{ MHz}$, $N = 3$, and varying J between 1 and 100 MHz, the quasibound states' lifetime ranges between $10^{-7} \text{ s} \leq \tau_{B\pm} \leq 10^{-3} \text{ s}$, much larger than the original bare lifetime $\kappa^{-1} \sim 10^{-8} \text{ s}$.

Increasing the hopping $J \gg g$, the polariton states transform into photonic states which are again described by a Lorentzian but with a linewidth that decreases only algebraically with N :

$$W_{B\pm} = \frac{2\kappa}{N+1} \sin^2[\pi/(N+1)] \sim \frac{2\pi^2 \kappa}{N^3}. \quad (15)$$

The exact linewidth $W_{B\pm}$ is plotted in Fig. 4(a) as a function of the effective hopping strength J/g together with the approximations at weak (14) and large hopping (15). In Fig. 4(b) we plot the photon occupation probability in the MM evaluated at the quasibound state energies, i.e., $n_{B\pm}^j = \langle \phi_k | a_j^{\dagger} a_j | \phi_k \rangle |_{\epsilon_k \approx \Omega_{B\pm}}$, for an array with 101 cavities. At weak hopping $J \ll g$, the excitation is localized in a narrow region around the central

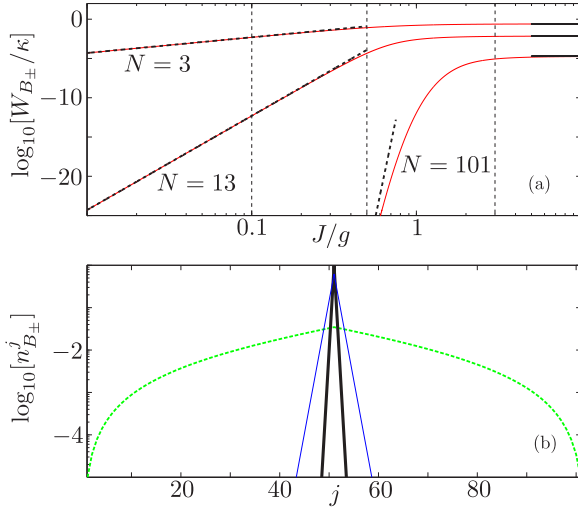


FIG. 4. (Color online) Panel (a) shows the exact linewidth of the resonances at $\Omega_{B_{\pm}}$, see Fig. 2, as a function of hopping strength for $N = 3, 13, 101$ cavities. The black dashed (solid) lines correspond to the asymptotic results for weak (large) photon hopping as described in Eqs. (14) and (15). Panel (b) shows the photonic occupation probability $n_{B_{\pm}}^j$ with site index j , evaluated at the quasibound state energies $\epsilon_k \approx \Omega_{B_{\pm}}$, for $N = 101$ cavities and $J/g = 0.1$ (black, thick), $J/g = 0.5$ (blue, thin), and $J/g = 3$ (green, dashed).

cavity (black, blue lines), while it delocalizes over the lattice at large hopping $J \gg g$.

Finally, we comment on the in-band resonances, e.g., the broad peaks appearing in Fig. 2(b) at $J/g = 0.5$. In order to do so, it is helpful to analyze the spectra in Fig. 3. Given the photonic and qubit degrees of freedom, we expect a total of $N + 1$ states of which $N - 1$ should be located inside the band. At small J , the central cavity mixes with the qubit to generate the (quasi-)bound states at $\omega_c \pm g$; for $N = 3$ no oscillator strength at the central cavity is left for the other two modes (this

remains true for all J) and we find a twofold degenerate state at $\Omega_n = \omega_c = \omega_q$, describing a pure photon and a photonlike state with small qubit weight, which does not mix. Increasing J , the pure photon state remains, the photon-dominated state transforms into a qubit-dominated state, and the (quasi-)bound states become photon dominated. In general, for odd $M = (N - 1)/2$, the spectrum still features a doublet of degenerate states at ω_c with the same properties as described above for $N = 3$; however, for $N > 3$, some weight of the central cavity is transferred from the quasibound states to the other in-band modes with increasing J . For even M , the two states near ω_c are both photonlike at small J and their mixing with the qubit with increasing J results in a splitting $2g\sqrt{2/(N + 1)}$. At small J , i.e., to order J , the resonances within the band appear in pairs, reducing the number of expected peaks by half. Finally, the transmission is exactly zero right at the qubit frequency due to the so-called dipole-induced reflectivity effect (DIR) [36], known from the single-cavity case. The observed (peak) structure in Fig. 2 is consistent with these considerations.

In summary, we have studied the single-photon transport properties of a one-dimensional coupled cavity array containing a single qubit in its center, and coupled to a transmission line supporting propagating photon modes with linear dispersion. For small hopping J , the transmission coefficient exhibits two high- Q polariton resonances associated with self-protected quasibound states of the qubit and photonic Bloch modes. We have derived a simple expression for the lifetime of these states and found that it increases exponentially with the number of sites, thereby realizing a regime of strong light-matter coupling. The proposed architecture is readily realizable within current state-of-the-art technology, e.g., in circuit QED [24,25,28,29].

We acknowledge financial support from the Swiss NSF through an Ambizione Fellowship (S.S.) and under Grant No. PP00P2-123519/1, the NCCR QSIT (M.B.), and the National Science Foundation under Grant No. DMR-1151810.

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